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SECONDARY BUCKLING IN HOLLOW RECTANGULAR COLUMN SECTIONS OF STEEL PLATES

by

Wendell Herman Nedderman

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Structural Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

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Dean of Graduate College

Iowa State College
1951

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NOTATION

x, y	Coordinate axes in plane of plate
ω	Deflection of plate at any point x, y
b, d	Width of plates in rectangular cross section
l	Length of plate
t	Thickness of plate
σ	Average applied load per unit area on member
σ_{cr}	Average applied load per unit area on member when any plate in member becomes elastically unstable
σ_{ult}	Average applied load per unit area on member at ultimate loading
σ_{yp}	Average unit stress in material when yield point of material is reached
P	Total load on member
P_{ult}	Total load on member at the ultimate loading
k	Dimensionless coefficient in Bryan formula
C	Dimensionless coefficient in Von Karman equivalent-width formula
E	Modulus of elasticity of material
E_m	Average load per unit area on member ÷ average of measured unit strains at four corners of the member
μ	Poisson's ratio for material
δ	Tangent modulus ÷ modulus of elasticity

ϕ	Angle of rotation of plate at corner of member
S	Shape factor
ϵ	Average unit strain
A	Total area in cross section of member
n	Scale-reduction factor

I. INTRODUCTION TO THE PLATE BUCKLING PROBLEM

For many years (38, 41) it has been recognized that a column may fail in one of two general ways: (1) failure by buckling or twisting, or both, with the entire column acting as a unit, and (2) failure by localized buckling of a component part of the column.

Lundquist (23) has defined primary column failure as any type of failure in which the cross sections of the column are translated, rotated, or both. He also has defined secondary column failure as any type of failure in which the cross sections are distorted in their own planes but are not rotated or translated. Barlow (5) has defined the critical stress for secondary buckling as the average unit stress over the whole cross section when any element of the section becomes elastically unstable.

The aeronautical engineering profession was quickly forced to recognize and to cope with the problem of secondary failure in thin-walled members of the type used in aircraft construction. However, the structural branch of the civil engineering profession has been relatively slow in meeting the problem of secondary buckling as it applies to structural steel of the proportions commonly found in the design of

structures. This lag may be a result of the fact that rolled steel sections have been ordinarily proportioned to preclude secondary failure. In general, built-up sections have been proportioned to prevent secondary buckling failure by staying within certain limits of design somewhat arbitrarily established by past practice. While this practice has, in the main, resulted in over-safe and highly understressed members with regard to secondary buckling, there have been many exceptions, some of notably catastrophic proportions like the Quebec Bridge failure (37).

In the 1930's considerable attention was given to the problem of secondary buckling of plate components of bridge towers (43, 44, 45). In the early 1940's the problem and theory of the elastic stability of component parts of compression members as applied to structural design began to receive the attention of the American civil engineering profession (32). At the close of World War II, the accelerated use of light-gage steel structural shapes further emphasized the problem of secondary buckling (29).

It should be recognized that most built-up sections and rolled shapes are composed largely of plates as component parts, and that under certain critical conditions, elastic instability of these plate elements may occur before the primary failing load of the column is reached. Thus in the interest of both safety and economy, the design of compression members should be investigated for both types of failure.

This thesis, dealing with hollow, rectangular column sections of mild structural steel, will be limited to the problem of secondary buckling. The objectives of this study are as follows:

(1) To determine the structural behavior of hollow, rectangular column sections of steel plates under axial compression insofar as secondary buckling is concerned.

(2) To indicate a method of design which can be used with safety and confidence in designing for secondary failure of members of this type OR to verify the applicability to members of this type of the methods now in use for other types of thin-walled construction.

(3) To indicate with what probable degree of dependability model analysis may be used to predict secondary buckling characteristics of larger sections when ordinary shop methods of fabrication are used for both models and prototypes.

The column sections used in this investigation are rectangular; however, each section is treated as four individual plates acting together. See Figure 1. Each plate is assumed to have a uniform axial load applied on its upper and lower edges; the unloaded edges are supported by the two adjacent plates. Basically, the problem is one of determining the elastic stability characteristics of a steel plate uniformly loaded on two opposite edges when the two unloaded

edges are given a degree of support encountered in practice, the effects of which need to be determined. Effects of initial distortions in the plate must also be determined experimentally.

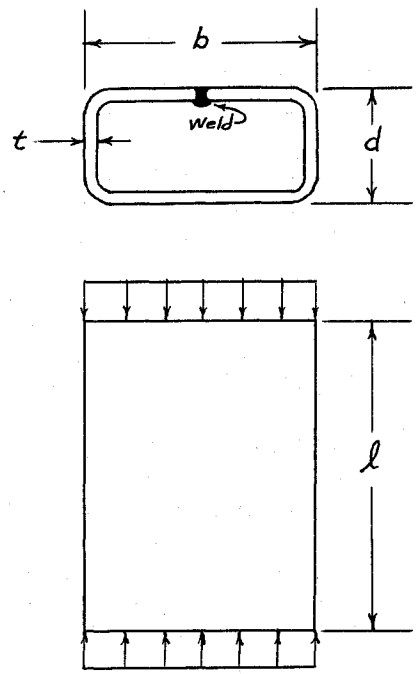

Fig. 1. General Shape of Cross Section and Type of Loading.

Since mild structural steel is the material used in this study, the idealized stress-strain curve shown in Figure 2

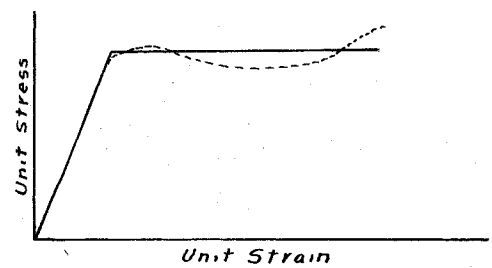

Fig. 2. Typical Stress-Strain Curve for Mild Structural Steel.

may be assumed (60). See Appendix E. Stresses above the idealized proportional limit have little significance other than that the useful limit of the material has been reached.

II. HISTORICAL REVIEW

A. Pertinent Analytical Studies

1. Small deflection theory

Timoshenko (48, 49) has given the equation for the deflection of a flat plate loaded axially and uniformly on opposite edges with no lateral movement of the edges as:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{t}{D}\left(\sigma_x \frac{\partial^2 w}{\partial x^2}\right); \quad D = \frac{E t^3}{12(1-\mu^2)}$$

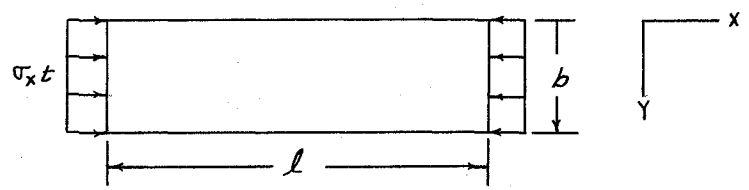

Fig. 3. Loading and Coordinates for Flat Rectangular Plate

Saint Venant is credited with the original generalized form of this equation. In many cases, the critical buckling stress can be found by satisfying this equation in addition to the boundary conditions set up by a given condition of edge support. This method is referred to as the "exact" method of solution. Bleich (6) has shown a generalization of this method to include inelastic buckling for materials which do not have a well defined yield point. Assuming quasi-orthotropic behavior of the material, the deflection equation

becomes:

$$\frac{\partial^4 w}{\partial x^4}\gamma + 2\frac{\partial^4 w}{\partial x^2 \partial y^2}\sqrt{\gamma} + \frac{\partial^4 w}{\partial y^4} = \frac{t}{D}\sigma_x \frac{\partial^2 w}{\partial x^2}$$

The quantity $\sqrt{\gamma}$ in the second term is somewhat arbitrarily assumed but seems to check fairly well experimentally. For mild structural steel, γ becomes unity.

However, the first presentation of a solution of the plate buckling problem was made by Bryan (7) in 1891. Bryan used the "energy" method which is an approximate solution but which is usually accurate enough for all practical purposes. The "approximation" element lies in the fact that the shape of the buckled curve must be assumed and is rarely the exact true shape of the actual buckled plate. The "energy" method can be used for any combination of types of edge support whereas in some cases the "exact" method cannot be used. In the analytical investigation of closed, thin-walled, rectangular sections such as are used in aircraft, Lundquist (23) found the "energy" method to agree with the "exact" method to within a fraction of one per cent.

Bryan presented the solution of a buckled plate with all edges simply supported as

$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\mu^2)\, b^2/t^2}$$

The dimensionless coefficient "k" is dependent on the length-width ratio ℓ/b of the plate. Timoshenko (48) and Hill (15)

have shown that for a given edge support, "k" remains constant for all long plates and changes only a very small amount if $\ell/b > 2$ or 3. Hill (15) and Maulbetsch (30) have shown that the degree of edge support at the loaded edges has no significant effect on the buckling characteristics of a long plate (i.e., $\ell/b > 2$ or 3). Timoshenko (49) has shown that theoretically a long plate with the edges clamped buckles into wave lengths about 0.7 times the unsupported width of plate and about 1.0 times the unsupported width of plate if the edges are simply supported.

It is customary (27, 59) to generalize the Bryan formula as

$$\sigma_{cr} = \frac{K\pi^2 E}{12(1-\mu^2)\, b^2/t^2}$$

where "k" depends on the ratio ℓ/b and on conditions of edge support. This formula indicates that the critical buckling stress of the plate is directly proportional to the modulus of elasticity for the material and inversely proportional to the square of the width-thickness ratio of the plate.

A solution by the Small Deflection Theory involves the following assumptions and limitations:

(a) Stresses are below the proportional limit. (For elastic buckling)

(b) Deflections are small (i.e., considerably less than the thickness of the plate).

(c) Middle plane of the plate does not stretch.

(d) Material is homogeneous and isotropic.

(e) Applied edge loading is perfectly axial and uniform.

(f) Edges of the plate remain straight with no lateral movement.

(g) Conditions of edge support are known.

(h) Initial distortion does not exist in the plate.

Assumptions (g) and (h) are of particular significance in limiting the usefulness of the theoretical solution. The degree of edge support and the magnitude of initial distortions in the plate exert considerable influence on the magnitude of the critical buckling stress of the plate (32, 34,48). Hence it is imperative that the effects of these factors be determined experimentally.

2. Large deflection theory

The solution of the plate problem for large deflections has been given by Von Karman, Timoshenko, and others (14, 22, 28, 48, 49). This theory is based on the same assumptions and limitations as the Small Deflection Theory except that:

(a) It recognizes stretching of the middle plane of the plate.

(b) It recognizes large deflections (i.e., several times the thickness of the plate).

(c) It recognizes that the applied edge load is no longer uniform after the critical buckling stress is reached.

(d) It assumes that the unloaded edges of the plate remain straight during buckling with large deflections.

According to Timoshenko (48), assumption (d) is satisfied in experimental tests if the plate buckles at stresses well below the proportional limit of the material. However, this assumption places a serious limitation on the theory if buckling occurs at relatively high stresses. Also, as before, the degree of edge support and the effect of initial irregularities in the plate must be determined experimentally.

3. Ultimate strength of a buckled plate

The ultimate strength of a plate may be many times the critical buckling strength (40, 57). This is true in the case of a wide thin plate which buckles at a stress well below the proportional limit of the material. In such a case, as the plate buckles, the load carried by the central portion of the plate is reduced, throwing an additional load upon the two supported edges of the plate which are not as free to buckle. The ultimate load will occur when the yield point is reached at the edges. Additional loading produces excessive deformations. The higher the stress at which the plate reaches the

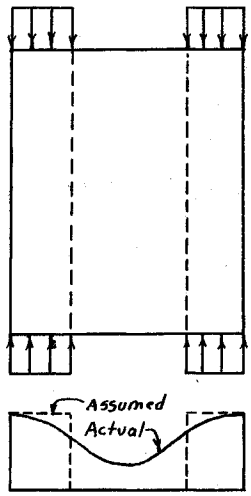

Fig. 4. Assumed Stress Distribution of Buckled Plate by the Equivalent-Width Concept.

critical buckling point, the smaller will be the difference between the critical buckling strength and the ultimate strength of the plate.

To determine the ultimate load of a plate, Von Karman (53) presented the "equivalent-width" concept. See Figure 4. Assuming simply supported edges, he found that

$$P_{ULT} = \frac{\pi}{\sqrt{3(1-\mu^2)}} t^2 \sqrt{E \sigma_{yp}}$$

Since true hinged-edge supports are impossible to realize in practice, the term $\frac{\pi}{\sqrt{3(1-\mu^2)}}$ is replaced by a constant C so that

$$P_{ULT} = C t^2 \sqrt{E \sigma_{yp}}$$

Von Karman (53) has shown that if the cross section were to buckle as shown in Figure 5a, then C = 1.9; if as in Figure

5b then C = 1.24. Actually the buckled shape is somewhere between these two extremes. Hence C must be determined experimentally. By this concept, the ultimate load is

5a 5b

Fig. 5. Assumed Shapes of Cross Sections by Equivalent-Width Method.

independent of the width and length of the plate and varies directly as t^2.

The equivalent-width method is particularly valuable in such cases as the design of aeronautical structures where very thin sections are used. Here the ultimate strength is often much larger than the critical buckling strength; visible buckling occurs long before the ultimate strength of the plate element is reached. Weight requirements demand a recognition of the full strength of the material. Also, visible "wrinkling" is not considered objectionable by either the designer or the user of the structure as long as safety requirements are met.

In the field of structural engineering, this method has been incorporated into some specifications (3) covering the design of light-gage steel shapes. However, its usefulness is extremely limited in the case of structural members of the proportions commonly found. For economy, short compression

members should be designed so that the entire cross section is fully effective up to the yield point of the material (59). Longer columns, subject to primary failure, are designed for primary buckling on the basis that the entire cross section is fully effective. Hence the small deflection critical buckling strength should be equal to or greater than the primary column strength for any compression member (31). Also, in the field of structural engineering visible wrinkling is not generally considered permissible by either the designer or the users of the structure (27).

4. Other analytical studies

Pai, Lundquist, and Batdorf (34, p. 8, 10, 14) made an analytical study of a square plate with simply supported edges under axial uniform edge loading to determine the effect of small deviations from flatness. Some of the pertinent observations and conclusions reached as a result of this study are as follows:

(a) When a plate with an initial deviation from flatness is subjected to compression, the deflections that result from application of load grow very slowly with the first increments of load but grow very rapidly as the critical load is approached.

(b) Since buckles begin to grow with the beginning of loading for a plate that has an initial deviation from flatness, there can be no buckling stress for an actual plate in a strictly theoretical sense; however, just as a defined yield stress has been

found useful for materials that have no actual yield stress, so a defined buckling or critical stress for a plate can convey much meaning to the engineer.

(c) The effective width for load-carrying capacity of a plate with an initial deviation from flatness is, at all values of stress, less than that of an initially perfect plate. The effective width for stiffness against further compression, however, is less than that of a flat plate at stresses below the theoretical critical stress but greater at stresses above.

These observations are in agreement with those previously expressed by Westergaard (54).

Hill (15) has prepared a series of charts for values of "k" in the Bryan formula for many types and combinations of edge support along the unloaded edges of a flat plate. The "energy" method is used.

Several excellent theoretical treatments have been made of the problem of buckling of a flat rectangular plate under a condition of elastic restraint against rotation along the unloaded edges. In square, closed sections with all sides

6a 6b

Fig. 6. Buckled Cross Sections of Hollow Rectangular Members.

of equal thickness (Fig. 6a) all plates become elastically unstable at the same time. Thus when any one plate becomes unstable, it is not offered any edge restraint against rotation by the two supporting plates which have also reached a state of instability. Under such a condition $\phi = \phi'$ and an equivalent hinged-edge support results. However, in the case of unequal widths (Fig. 6b), when the wide plates approach instability, the narrow plates are still stable and offer a restraint against rotation to the edges of the wider plates. In this case $\phi > \phi'$. This results in a higher value of "k" in the Bryan formula when computing the critical buckling stress of the wider plates.

Lundquist and Stowell (27) have summarized the values of "k" in the Bryan formula in chart form for flat rectangular plates for the condition of elastic restraint against rotation along the unloaded edges. The "exact" and "energy" methods are used. For the same type of edge conditions, Bleich (6) has determined a method for finding the "coefficient of restraint" and finally "k" for the Bryan Formula. This simplified method gives results which are slightly in error but which are on the conservative side of those given by the "exact" method. This same problem was approached by Kroll, Fisher, and Heimerl (20) who used the moment distribution method to determine "k" for the Bryan Formula.

Heck and Ebner (13) have presented an extensive summary of all important work published up to 1935 on the design of thin-walled structures principally as applied to airplane design but applicable to all thin-walled members.

Numerous other variations of, and additions to, the analytical treatment of the buckled plate have been presented. However, all are based on the fundamental groundwork previously mentioned.

B. Pertinent Experimental Investigations

The occurrence of secondary buckling in compression members has been observed in experimental work for many years. Some of the first published accounts of experiments involving localized buckling of compression members (17, 38) were made in 1850 by Clark who worked withthin tubular members and Fairborn who inquired into the strength of wrought-iron plates.

However, only in relatively recent years have tests been performed which are of special significance insofar as plate elements of the materials and types of construction used today are concerned. Some of the significant experimental tests on the behavior of a flat plate loaded axially and uniformly on two edges will be discussed.

Between 1918 and 1935, the United States National Bureau of Standards published the results of several tests on the buckling characteristics of plate elements of compression

members (11, 17, 43, 44, 45, 52). Two of these which are of special significance are the tests on full-size column web plates of the Delaware River Bridge (17) and the tests on one-half size models of the steel chord members of the Bayonne Bridge (45). In the Delaware River Bridge tests, silicon-manganese steel plates were enclosed on all edges by heavy angles and closely riveted. The members of the Bayonne Arch were made of carbon-manganese steel; the cross sections were the double-box type with diaphragms and continuous walls. The cover plates were attached at each unloaded edge to an angle with a single row of rivets. The b/t ratio of the cover plate between lines of rivets was 45. The cover plate began to buckle over its entire length at 38,000 psi even though the yield point of the material was 55,000 psi.

Wyly (59, p. 73) has summarized the conclusions to be drawn from the Delaware River bridge and Bayonne bridge tests as follows:

1. Boundary conditions.

   (a) Only a simply supported edge condition may be safely assumed in designing. It is seldom possible to secure clamped edges. The nearest approach shown experimentally was the 3/8-inch web enclosed between two heavy angles on each edge and the whole closely stitch-riveted together. The angles were 3/4-inch thick; in short, the total thickness of the two enclosing angles was four times that of the web. Even here the restraint was not equal to the clamped condition.

(b) The angles of other material supporting the edges must be stiff enough to prevent displacement of the edges of the plate.

(c) The spacing of the rivets connecting the edges of the plate to supporting angles or other stiffening elements should not exceed about ten times the plate thickness for alloy steel.

2. Effect of initial kinks or waves in plate.

(a) The effect of initial kinks or short waves in the plate will be to lower the buckling strength materially. For the 5/8-inch plates in the Delaware tests, a short wave of 1/4-inch double magnitude reduced buckling strength about 23 per cent.

(b) The thinner the material the greater is the probable effect of such kinks. Probability of initial kinks increases with decreasing thickness. Magnitude of kink compared with thickness of plate becomes greater with decreasing thickness.

3. Effect of local buckling.

It is unsafe and uneconomical to use an element with deficient buckling strength even though only part of the section is counted. The effect of local buckling on one element is to initiate local failure of the remaining elements.

In 1931 Schuman and Bach (40, p. 536) published the results of a series of tests on flat rectangular plates. The plates were uniformly loaded on opposite edges and the unloaded edges were supported in V-grooves of a rigid frame. All the plates were 24 inches long, varied in width from 4 to 24 inches, and varied in thickness from 0.015 to 0.095 inches. Duraluminum, stainless iron, monel metal, and nickel plates were used. Some of the conclusions reached are as follows:

1. For the plates of this investigation, plates loaded in the direction of rolling, buckling occurred at loads less than the Bryan load.

2. Except for cases noted below, the plates carried loads above the Bryan load. The wide and thin specimens in particular carried much greater loads. The stainless iron, Monelmetal, and nickel plates 4 inches wide carried less than the Bryan load when their thicknesses equaled or exceeded 0.06 inch.

3. The maximum load carried by a plate depended far more upon the thickness than upon the width of the plate unless the plate was narrow. (In this work less than four inches.)

In 1940 Holt (16, p. 215) published the results of tests on 17 members of high-strength aluminum alloy and of mild steel. These sections were of various shapes composed of angles, plates, and channels. Each section had a slenderness ratio of about 40 and a cross sectional area of about 7 to 8 square inches. Some of the conclusions are as follows:

1. For low loads the shape of the cross section has no significant effect on the distribution of the stress; but for the higher loads, the specimens with thin outstanding plates have a pronounced variation in the stress.

2. The shape and proportions of the cross section have a significant effect on the ultimate strength and the type of failure, whether by bending as a unit or by local buckling.

3. The type of failure, whether by local buckling of a component part or by bending as a unit, can be predicted as the type of failure corresponding to the least of the strengths computed by the various equations for critical stress and column strength.

4. In order to avoid failure of short columns (l/r ratio about 40) by local buckling at a

load less than that predicted for the column acting as a unit, the width-to-thickness ratio of outstanding plates and legs of angles should not exceed about 12 and the ratio of width-to-thickness of cover plates with a row of rivets near each edge should not exceed about 40.

5. Local buckling will not always result in the collapse of the entire member, but may be only a wrinkling of the buckled part.

In 1946 Winter (57, p. 551) published results of an extensive series of tests on light structural shapes from sheet steel. The members studied were of two types: (a) members with compression flanges both of which were edge stiffened (such as top flanges of inverted U-beams), and (b) members with compression flanges with only one longitudinal edge stiffened (such as either half of a flange of an I-beam). The members of the first type tested varied from 3.4 to 10.1 inches in width, from 0.0237 to 0.1478 inches in thickness, from 1.49 to 8.00 inches in depth, and from 14.3 to 170.0 for values of b/t ratio. The equivalent-width concept was used and working formulas for design purposes were derived. An important conclusion reached for flanges stiffened along both edges is as follows:

> Flanges with values of b/t to about 25 were found to fail by simple yielding, the full area of the flange being effective -- that is, the stress is of uniform magnitude across the width. For flanges with larger values of b/t, deflections and ultimate loads were found to err on the dangerous side.

Wimer (56) conducted a series of tests on hollow 18-8 stainless steel sections with about the same range of b/t values as used by Winter. The results closely checked those obtained by Winter.

III. EXPERIMENTAL INVESTIGATION

A. Materials and Test Members

Each member was fabricated under ordinary shop methods from a single hot-rolled, mild, steel plate. Each plate was cold-bent to the proper shape and joined with a single longitudinal weld along the middle of one side to form a closed rectangular section. The welded bead on the external surface of the member was ground level to the wall of the member. Because the corners of the members were cold-bent, strain developed in the plastic range in a direction perpendicular to the longitudinal axis of the member. No attempt was made to relieve this strain, hence this study is based upon the behavior of plates under conditions which can prevail in practice.

The length of each member was made short enough to eliminate primary column action yet of sufficient length to indicate the secondary buckling characteristics of a long plate. In addition, effects of torsional instability were excluded since the sections were closed and had double axes of symmetry (19). Thickness of plates was calipered and the variations were within acceptable limits.

The ends of each member were milled in a Universal milling machine, then "spotted in" by hand filing to give

ends as flat as practicable. This was done by using a very thin coat of Persian Blue painting oil on the surface of a heavy, ribbed plate which was known to be almost perfectly flat.

Four series of test members were made up as shown in Table I. Series A was designed to indicate the degree of accuracy of model analysis using ordinary shop methods of fabrication. Although assumed to be geometrically similar under these conditions, actually the members were not exactly so. This departure from geometrical similarity was due to the different degrees of initial irregularity in the plates, varying radii of curvature of the corners, non-geometrical similarity in sizes of welds, effects of heating, and other factors of lesser significance. Series B was designed to generalize this study by using rectangular as well as square sections. Series C was designed as a check on other members and to show whether a plate with a length-width ratio of two behaves substantially the same as longer plates. Series D was provided to extend the range of values of the b/t ratio and to provide checks on the other members. The b/t ratio range of 24 to 80 was felt to be representative of the range most likely to be used in the structural engineering field.

Table I Geometric Properties of Test Members

Member	Nominal Dimension b in.	Nominal Dimension d in.	Nominal Dimension t in.	Gage	Actual Dimensions b in.	Actual Dimensions d in.	Actual Dimensions t in.	Actual b/t Ratio	Milled Length in.	Weight lb.	Cross Section Area - sq. in. By Weight	Cross Section Area - sq. in. By Measurement	Average Radius Corners - in. Compute	Average Radius Corners - in. Measure
A-1	10.0	10.0	0.250	—	10.1	10.1	0.238	42.3	20.17	51.73	9.05	9.36	0.75	0.65
A-2	8.4	8.4	0.209	5	8.2	8.2	0.203	40.4	16.95	30.83	6.44	6.52	0.24	0.40
A-3	5.4	5.4	0.135	10	5.2	5.2	0.131	39.7	10.95	8.18	2.64	2.68	0.16	0.50
A-4	3.0	3.0	0.075	14	3.0	3.0	0.077	39.0	6.28	1.55	0.87	0.90	0.21	0.25
B-1	10.0	8.0	0.250	—	10.1	8.1	0.241	41.8/33.6	20.17	46.81	8.19	8.51	0.81	0.85
B-2	10.0	6.0	0.250	—	10.0	6.0	0.247	40.5/24.3	20.22	42.23	7.37	7.70	0.77	0.75
B-3	10.0	6.0	0.209	5	10.1	6.0	0.200	50.5/30.3	20.16	34.60	6.06	6.26	0.57	0.60
B-4	10.0	6.0	0.135	10	10.0	6.0	0.138	72.5/43.5	20.15	23.57	4.13	4.34	0.89	0.90
B-5	10.0	6.0	0.135	10	10.1	6.1	0.139	72.7/43.9	20.29	24.39	4.23	4.51	0.90	0.80
C-1	10.0	10.0	0.250	—	10.1	10.1	0.270	37.4	49.83	145.53	10.30	10.70	0.87	0.80
C-2	10.0	6.0	0.250	—	10.1	6.1	0.267	37.8/22.8	49.84	111.41	7.89	8.34	1.00	0.90
C-3	5.4	5.4	0.135	10	5.4	5.4	0.136	39.7	48.65	39.90	2.89	2.87	0.53	0.60
D-1	10.0	10.0	0.120	11	10.1	10.1	0.126	80.1	20.14	28.20	4.94	4.96	0.34	0.50
D-2	10.0	10.0	0.135	10	10.1	10.1	0.137	73.7	20.21	30.13	5.26	5.46	0.79	0.70
D-3	10.0	10.0	0.135	10	10.1	10.1	0.133	76.0	20.09	28.92	5.07	5.37	0.68	0.75
D-4	10.0	10.0	0.164	8	10.1	10.1	0.157	64.4	20.06	34.72	6.12	6.23	0.39	0.50
D-5	10.0	10.0	0.209	5	10.1	10.1	0.200	50.5	20.19	44.04	7.70	7.59	0.55	0.55
D-6	8.0	8.0	0.250	—	8.0	8.0	0.247	32.4	16.19	33.33	7.27	7.63	0.96	0.95
D-7	6.0	6.0	0.250	—	6.1	6.1	0.248	24.6	12.13	18.45	5.37	5.79	0.97	0.90

B. Equipment Used

All tests of main members were made with a 300,000 pound Universal hydraulic testing machine. Electric strain gages (SR-4, Type A-1) were used to determine stress-strain distribution in the member. The Baldwin-Southwark strain scanning recorder and the Baldwin-Southwark 48-gage automatic switching unit were used to read and record the strain readings. The plate contour meter was designed and built to provide a highly reliable reference plane from which to measure the irregularities of the unloaded plates and to measure the horizontal deflections of the plates under various increments of loading. Two Ames dials were fastened to the top of the contour meter to indicate any relative movement between the contour meter and the testing machine. To accommodate the facilities of the contour meter, four 2-inch thick steel plates were placed between the lower end of the member and the bed of the testing machine. Each plate was milled on both sides. For description and details of contour meter see Appendix D. The contour meter was plumbed with the use of a plumb bob by measuring the distances to two perpendicular finished surfaces on the vertical assembly of the contour meter.

Fig. 7. General View of Test Member and Testing Equipment.

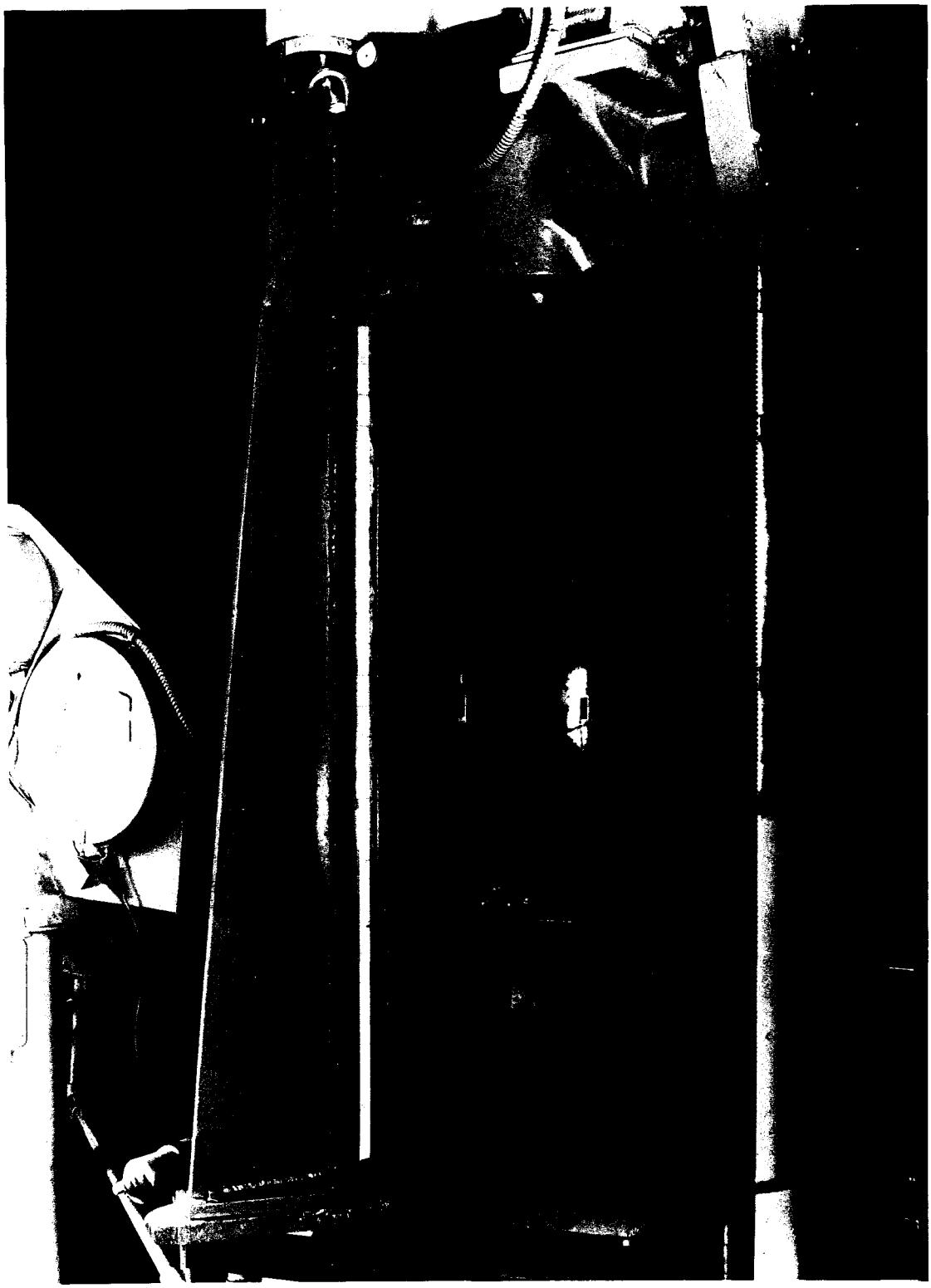

Fig. 8. General View of Plate Contour Meter in Operation on Testing Machine.

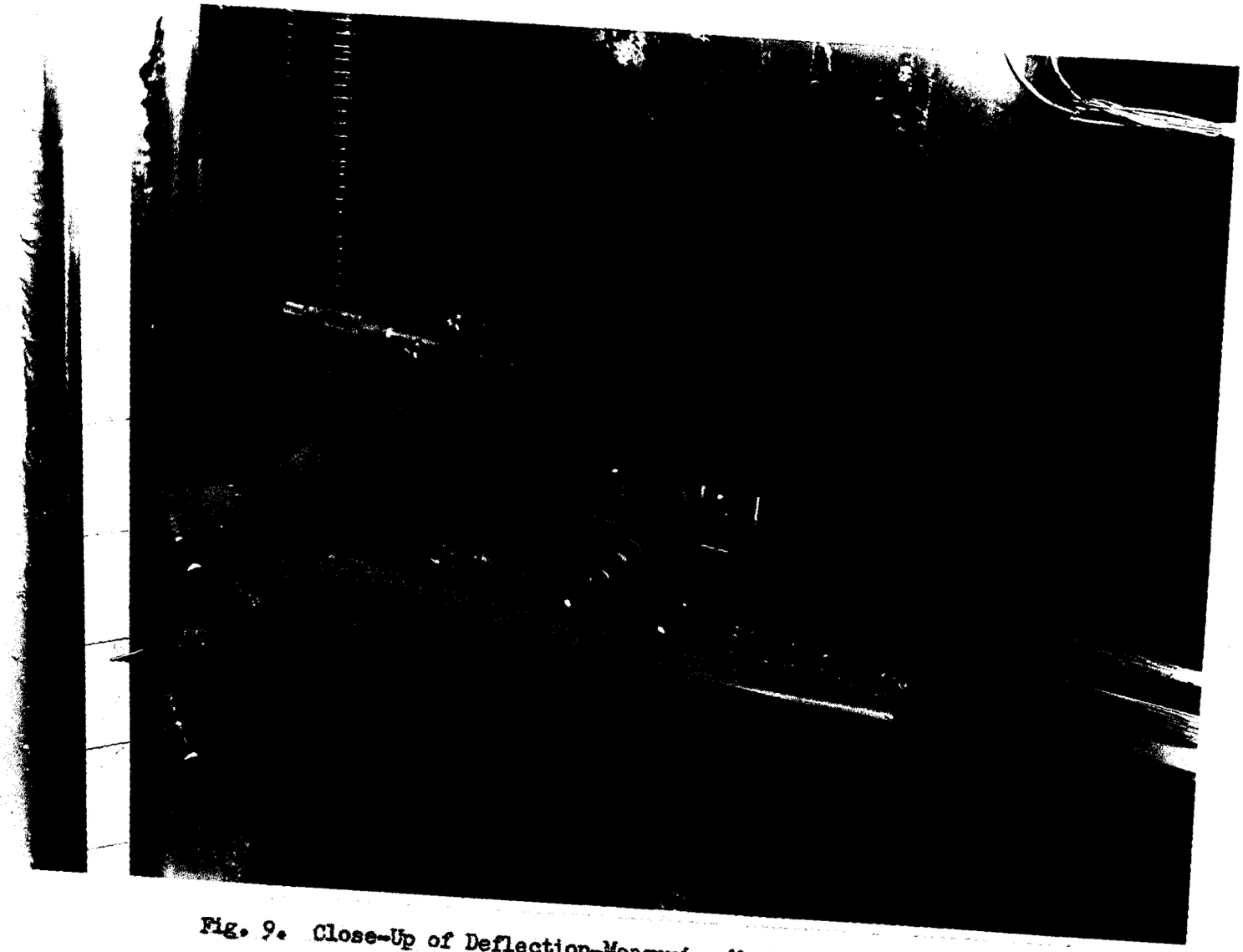

Fig. 9. Close-Up of Deflection-Measuring Mechanism of Contour Meter.

C. Laboratory Procedure

Since the test members were shop-fabricated and had varying radii of curvature at the corners, it was extremely difficult to determine the true cross sectional area by direct measurement. Hence the area of each member was computed in terms of its weight, using the weight of the member, milled length, and density of the material.

All members were axially loaded and tested with flat ends. The load was applied at the upper end of each member through a spherical-bearing head to eliminate the effect of any small non-perpendicularity of the planes of the ends of the member relative to the longitudinal axis of the member.

Each member was carefully centered and the spherical-bearing head rotated and adjusted just before coming in contact with the member. Slight adjustments were made in the position of the member to equalize as much as practicable the stress-strain distribution at the four corners under a small load. Despite the careful laboratory procedure and the careful "spotting-in" of the ends of the test members, the best that could be consistently obtained in equality of measured strains at the four corners of a member was a range of about ± 10 per cent of the average measured strain at the four corners.

Two SR-4 strain gages were placed on each corner at mid-height of each member to determine the stress-strain

distribution and general behavior of the member.

Preliminary to the main tests, two tensile test coupons taken from each stock of material used in fabricating the main members were tested to determine the modulus of elasticity and the yield point. The coupons conformed to ASTM standards. See Appendix E.

Also preliminary to the main tests, a check test was made using the SR-4 scanning recorder, the Type L SR-4 strain indicator, and Huggenberger tensometers on the same tensile coupon under identical loading conditions. See Appendix F.

Since deflection readings could be taken on only two adjacent sides of each member, as a general rule the side most visibly irregular was chosen as one of the sides to be measured. Profile readings were taken along the vertical centerline, along the two edges, and in some cases along two intermediate lines, on each of the two sides measured. Deflection readings were taken for zero loading, at various increments of loading, and after failure. At each increment of loading, strain readings were taken at the four corners of the member.

The profile lines on each exterior surface of a member were designated by numerals 1 to 5 and by 1 to 3 where intermediate profile lines were omitted. Points on each profile were designated in sequence from top to bottom by letters a, b, c, etc. Roman numerals were used to designate the four sides of each member.

Since the deflection of the plates is an important element in the study of the buckling of plates, it was necessary to determine the degree of reliability of the measuring device and technique. Therefore, profile readings were taken on one side of an unloaded member; the following day, without disturbing the set-up, the profile readings were repeated. Approximately 50 per cent of the readings were identical and the remaining readings were within ±1/1000 inch of those taken the previous day. This consistent performance of the contour meter for measuring the displacement of points on the plates established confidence in the subsequent readings taken for various increments of loading.

IV. PRESENTATION AND DISCUSSION OF RESULTS

A. Comparison of Theoretical and Test Values of Critical Buckling Loads; Derivation and Comparison of Working Curves

The term "elastic buckling failure" of a member will be used to denote failure which is initiated by critical buckling of a plate before the yield point is reached at any corner of the member. Likewise, the term "inelastic buckling failure" of a member will be used to denote failure which is not caused primarily by elastic buckling of a plate but failure which is initiated when the yield point is reached at one or more corners of the member and the failure is by plastic buckling of the material. In this study the "critical buckling load" on a plate will be defined as the average unit load applied to a member, of which the plate is a part, when any element of the member becomes elastically unstable.

Of the 19 members tested, B-4, B-5, D-1, D-2, and D-3 failed by elastic buckling. See Table II and Table III. The member D-4 was unintentionally loaded to the ultimate before the desired data were taken. The experimentally-determined critical buckling load of each member which failed elastically was found by first plotting the centerline profiles of two

Table II. Comparison of Test and Theoretical Critical Buckling Loads for Members Which Failed by Elastic Buckling

Member	b/t	Theoretical		Test
		k for Bryan Formula	Bryan σ_{cr} psi	σ_{cr} psi
B-4	72.5/43.5	5.1	26,100	24,000
B-5	72.7/43.9	5.1	26,100	23,000
D-1	80.1	4.0	16,800	17,000
D-2	73.7	4.0	19,800	20,000
D-3	76.0	4.0	18,700	18,000

Table III. Comparison of Theoretical Critical Buckling Loads and Test Ultimate Loads for Members Which Failed by Inelastic Buckling

Member	b/t	Theoretical		Test
		k for Bryan Formula	σ_{cr} psi	σ_{ult} psi
A-1	42.3	4.0	60,200	32,700
A-2	40.4	4.0	66,400	31,600
A-3	39.7	4.0	68,600	38,300
A-4	39.0	4.0	70,800	34,800
B-1	41.8/33.6	4.7	72,400	33,600
B-2	40.5/24.3	5.1	83,800	35,400
B-3	50.5/30.3	5.1	54,700	28,000
C-1	37.4	4.0	77,400	--
C-2	37.8/22.8	5.1	97,000	34,000
C-3	39.8	4.0	68,200	29,100
D-4	64.4	4.0	26,100	24,500
D-5	50.5	4.0	42,400	27,600
D-6	32.4	4.0	102,800	39,300
D-7	24.6	4.0	178,000	46,800

sides of the member for several increments of loading, then a load-deflection curve for a critical point from each of the two centerline profiles was used to determine the average applied unit load at which each plate approached instability. For example, see Figure 10. The load at which there was a sudden large increase in the rate of buckling was chosen as the test critical buckling load. The maximum measured load deflection at the critical buckling load was about one-third of the plate thickness for the members of square cross section which failed by elastic buckling and about one-half of the plate thickness for the members of rectangular cross section (b/d = 0.6) which failed by elastic buckling. (See Table IV.)

Many pertinent observations may be made from the test results for members of square cross section as shown in Figure 11. Curve A shows the critical buckling loads for plates under the idealized assumptions used in the derivation of the Bryan formula. Curve B was suggested by George E. Beggs in a discussion of a paper by Timoshenko (51) as a measure of the ultimate buckling resistance of commercial web plates of structural steel used in cellular compression members having one or more squares in their sections. Rolled steel plates of this type were observed by Beggs to have initial irregularities of a magnitude of about b/350 from the crests to the dents of the initial buckles. Curve C is the proposed curve for the buckling resistance of the test members. In the b/t

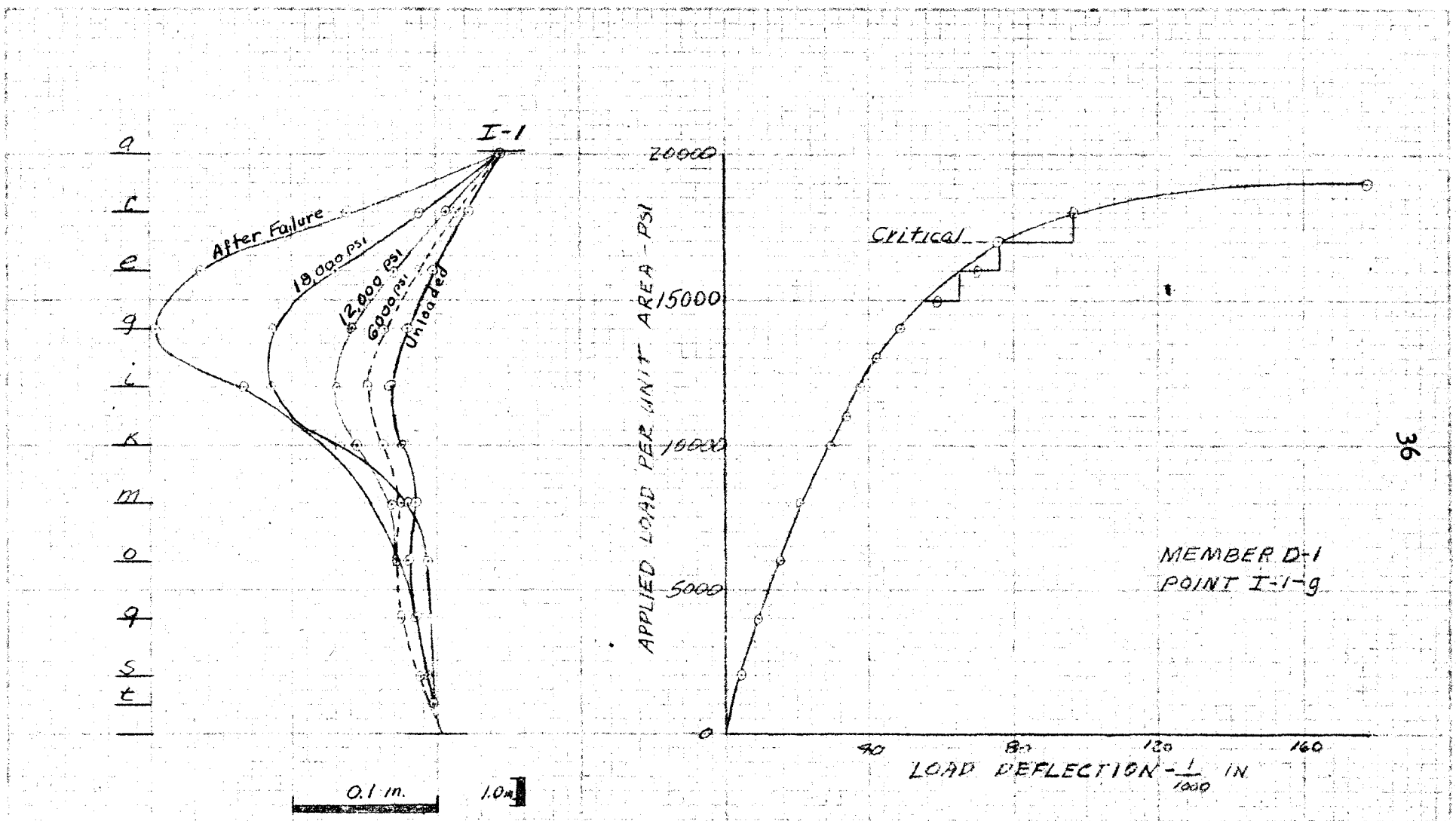

Fig. 10. Typical Load-Deflection Curve for Finding the Critical Buckling Load of a Plate.

Table IV. Relationship Between Plate Thickness and Maximum Measured Load Deflection at the Test Critical Buckling Loads for Members Which Failed by Elastic Buckling

Member	b/t*	Plate Thickness in.	Maximum Measured Load Deflection at σ_{cr} in.	Maximum Deflection at σ_{cr} ÷ Plate Thickness per cent
B-4	72.5/43.5	0.138	0.076	63
B-5	72.7/43.9	0.139	0.064	48
D-1	80.1	0.126	0.050	37
D-2	73.7	0.137	0.046	33
D-3	76.0	0.133	0.040	30

*The first value of b/t is for the long side and the second for the short side of the cross section.

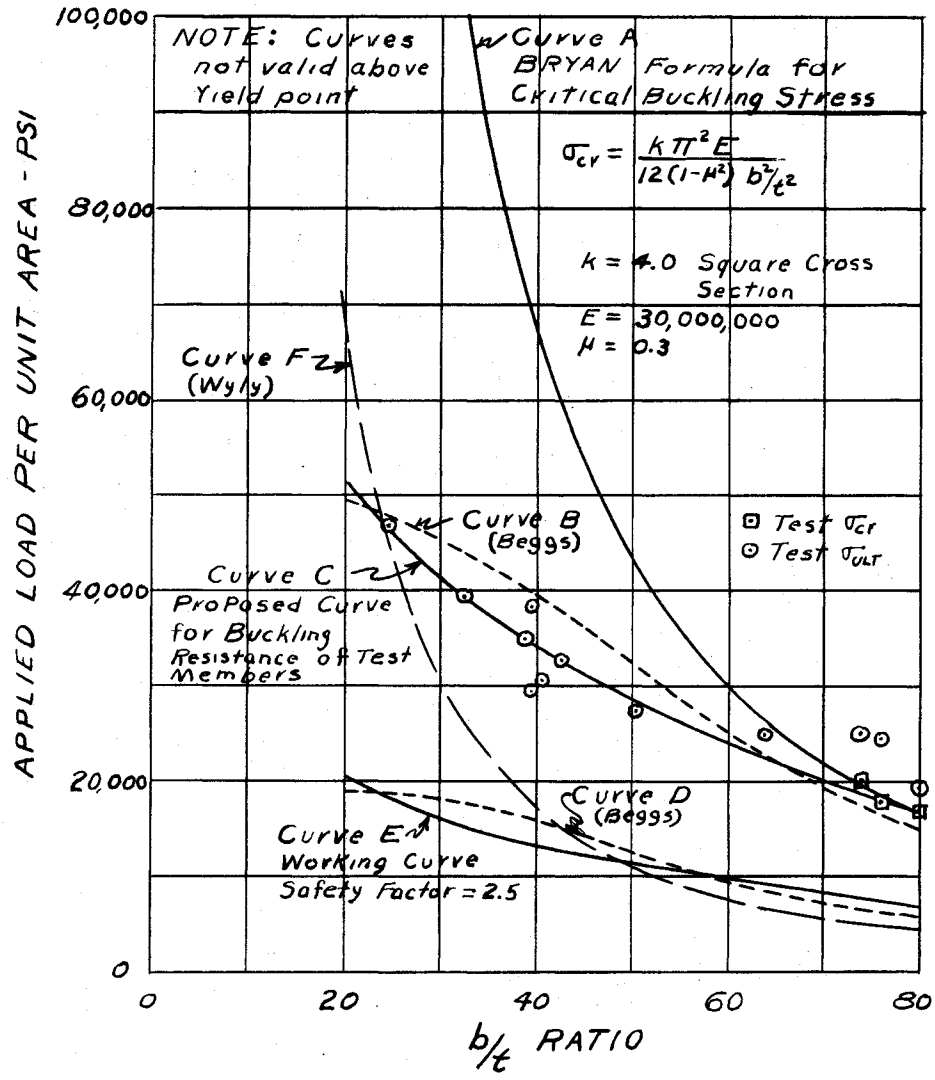

Fig. 11. Plotted Theoretical and Test Results Including Working Curves for Members of Square Cross Section.

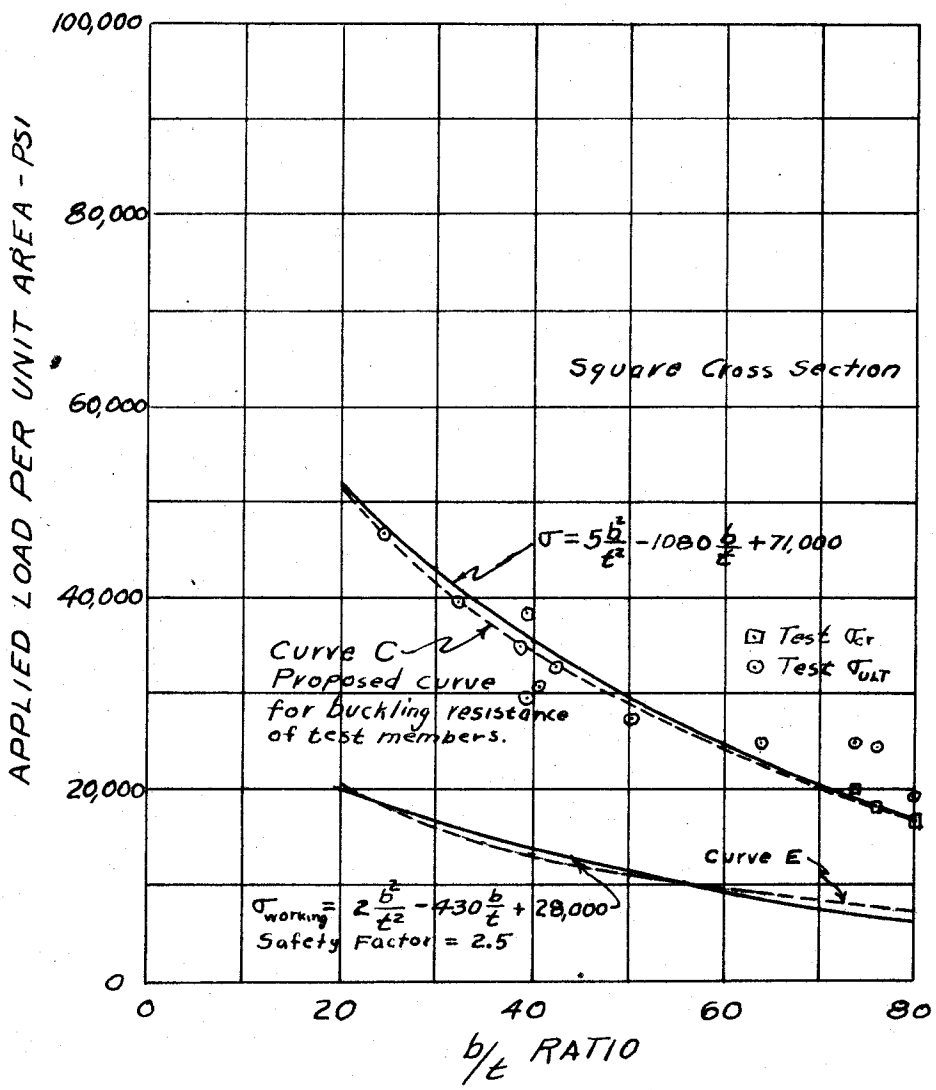

Fig. 12. Plot of Proposed Equations for Buckling Resistance Curve and Working Curve for Members of Square Cross Section.

ratio range of 70-80, elastic buckling failure occurred; such action probably also occurred in the b/t range of about 60 to 70. In this range of b/t ratios, the test critical buckling loads were used in lieu of the test ultimate loads as a measure of the useful strength of the members because excessive deflections occurred soon after the test critical buckling loads were reached. However, for a b/t ratio of less than about 60, the departure from idealized conditions which are present in an actual member under an actual load did not allow the members to reach the elastic buckling loads predicted by the Bryan formula. Instead, inelastic buckling failure predominated. In this range of b/t ratios, the ultimate strength was used as the useful strength of the members since appreciable load deflections (considerably less than the thickness of the side of the members) did not occur until the ultimate strength of the members was reached. See Appendix A. The amplitude of the initial waves in the test members was higher (up to b/50 for member A-4) than the initial double amplitude of b/350 in the plates observed by Beggs in proposing curve B. This may, in part, account for curve C being generally lower than curve B. Curve D was proposed by Beggs as a working curve and was derived by applying a safety factor of 2.5 to curve B. Curve E is a proposed working curve for the members tested and was similarly derived by applying a safety factor of 2.5 to curve C. The safety factor of 2.5 was somewhat

arbitrarily chosen. Actually, the exact factor of safety to be applied to curves B or C is dependent on the judgment of the designer. Wyly (54) has proposed curve F as a working curve for structural steel plates in riveted built-up sections in which the entire width of the plate is used to compute the b/t ratio and hinged-edge conditions are assumed. This curve was derived by applying a factor of safety of 4 to the Bryan formula (Curve A) to allow for the uncertainty of the degree of edge support of the plate and the initial irregularities in the plate. For sections designed to meet minimum specification requirements rather than stress requirements, a thinner plate may be allowed depending on the actual stress carried by the member. To determine the allowable b/t ratio for a given unit stress σ in the member, the curve F is frequently expressed in the form,

$$b/t = \sqrt{\frac{27,000,000}{\sigma}}$$

A comparison of the three proposed working curves (D, E, and F) shows that while the Beggs Curve D and the test Curve E are in general agreement, the Wyly Curve F does not appear to be consistently justified for the members considered in this investigation. In view of the divergence of curves A and C, it seems more justifiable to apply a reasonable factor of safety to a curve representing the actual behavior of the members than to the curve representing the theoretical behavior of the members.

In the b/t ratio range of 70 to 80, the test critical buckling loads checked very closely with the theoretical values using k = 4.0 in the Bryan formula. However, it should be noted that k = 4.0 has been theoretically computed for long plates with perfectly-hinged edges assumed. Such edge conditions are closely approximated in hollow members of square cross section having thin plates and right-angle corners. The test members had an average radius of curvature (incidental to fabrication) varying from 0.35 inches to 0.90 inches at each corner. This very likely changed the true theoretical "k" value. Also, the measured width of plate "b" used to compute the b/t ratios included a curved plate element at each edge of the plate. For the degree of edge support offered the plates of a member of this type of construction, the increase in buckling strength provided by the curved-edge plate elements appears to offset the decrease in strength caused by the initial irregularities in the unloaded plate and other slight departures from ideal conditions. The cumulative result seems to be that k = 4.0 may be safely used in the Bryan formula for a range of b/t ratios from 70 to 80.

Test results for the members of rectangular cross section with a b/d ratio of 0.6 are shown in Figure 13. The theoretically-computed value of k = 5.1 for thin-walled, hollow, rectangular cross sections with a b/d ratio of 0.6 was used (23). Curve A shows the Bryan critical buckling load

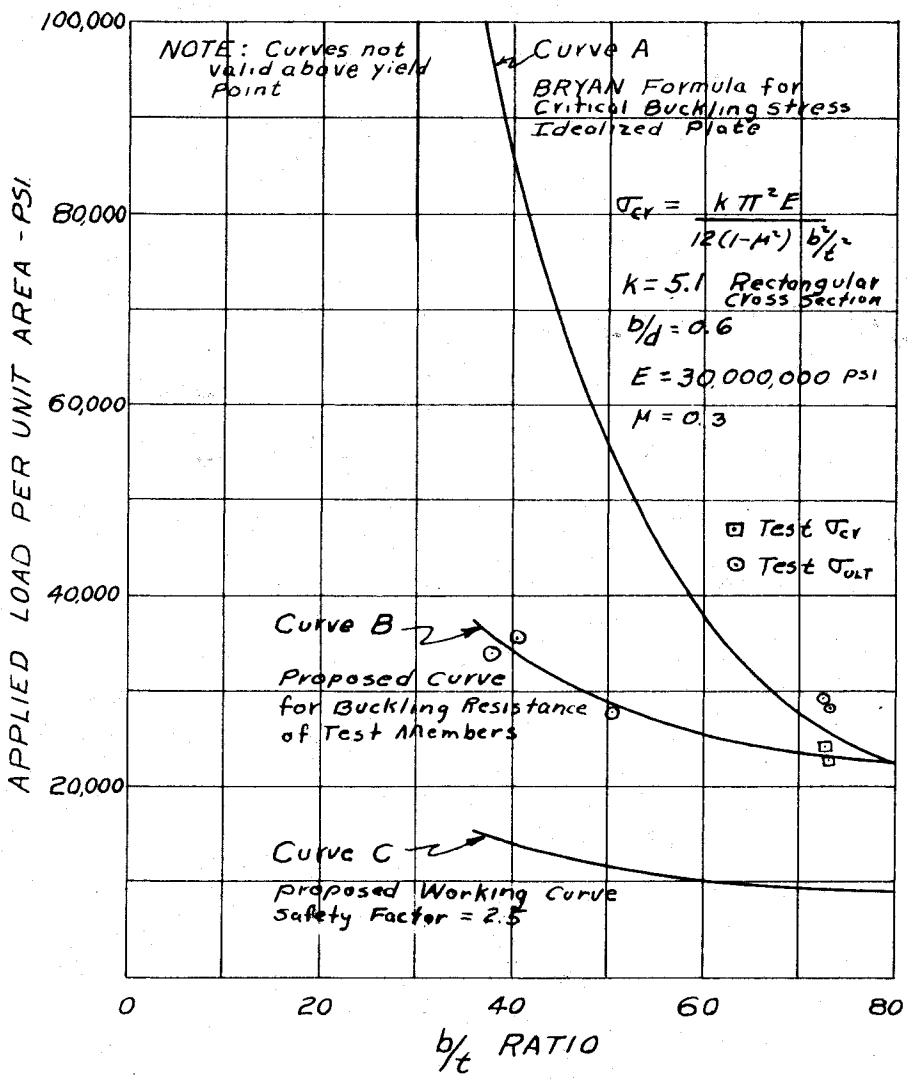

Fig. 13. Plotted Theoretical and Test Results Including Working Curve for Members of Rectangular Cross Section With a 0.6 b/d Ratio.

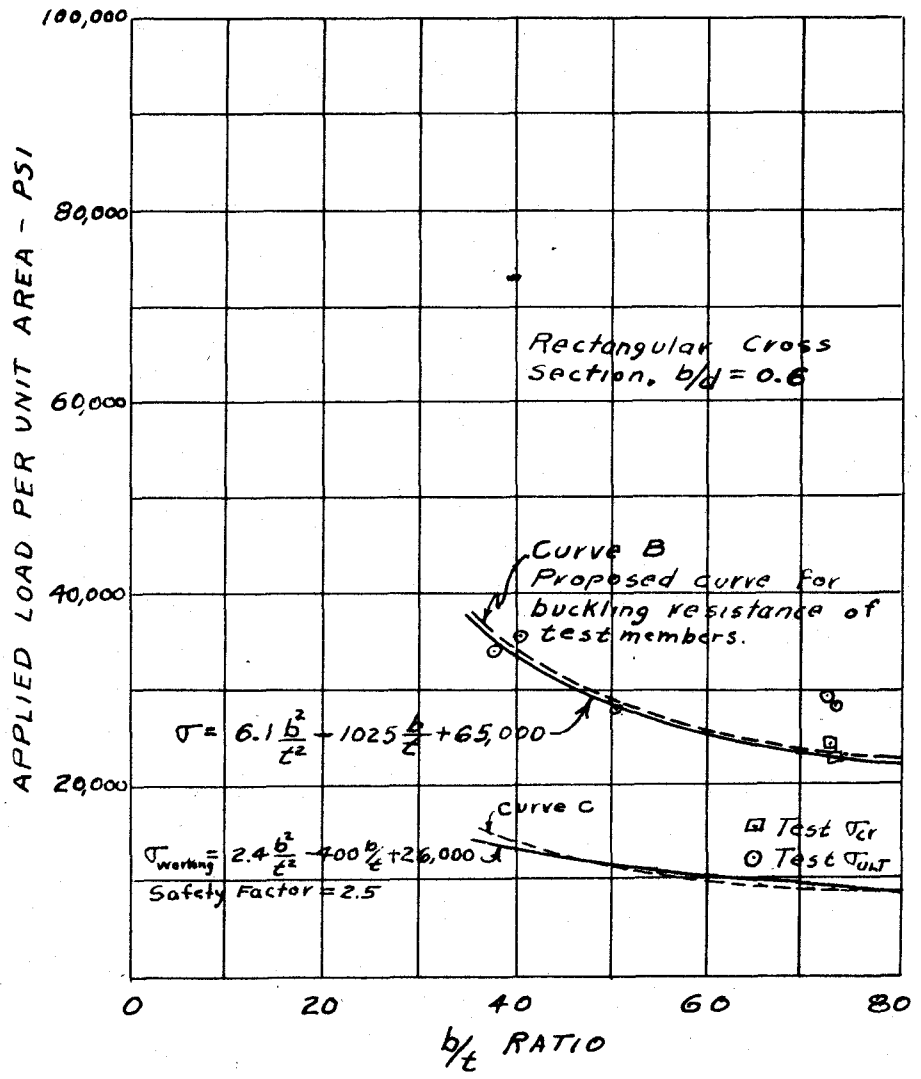

Fig. 14. Plot of Proposed Equations for Buckling Resistance Curve and Working Curve for Members of Rectangular Cross Section With a 0.6 b/d Ratio.

under idealized conditions for the wide plates of each rectangular cross section. While five test members are not enough for conclusive results, curve B is presented as an indication of the buckling resistance which may be expected from members of this type. Curve C is a proposed working curve derived by applying a safety factor of 2.5 to curve B. In the range of b/t ratios from 35 to 60, working curve C, Figure 13, is for all practicable purposes the same as the working curve E, Figure 11, which was proposed for members of square cross section. There was only a slight difference in the ultimate buckling loads of the square and rectangular cross sections which failed by inelastic buckling (b/t ratio up to about 60). However, in the elastic buckling range (b/t ratio of about 60 to 80) the working stresses of the rectangular sections are higher as would be expected since elastic restraint against rotation is provided at the unloaded edges of the plates of the members.

In Figure 12 is shown the equation of a curve which closely approximates the curve C of Figure 11, the proposed curve for buckling resistance of the test members of square cross section. The equation suggested is,

$$\sigma = 5\frac{b^2}{t^2} - 1080\frac{b}{t} + 71000$$

This equation should be limited to a b/t range of 20 to 80 and is not valid for σ above the yield point of the material.

This equation may be converted to a working equation by applying the desired factor of safety.

In Figure 14 is shown the equation of a curve which closely fits the curve B of Figure 13, the proposed curve representing the probable behavior of the test members of rectangular cross section (b/d = 0.6) used in this investigation. The equation suggested is

$$\sigma = 6.1 \frac{b^2}{t^2} - 1025 \frac{b}{t} + 65000$$

until determined otherwise by tests, this equation should be limited to a b/t range of 40 to 80.

B. Stress-Strain Relationships of Test Members

The relationship between the average of the measured longitudinal strains at the four corners and the computed applied load per unit area of each member is shown in Appendix C. The relationship between the test ultimate buckling loads and the proportional limits of the stress-strain curves for the test members is shown in Table V. The proportional limit of the stress-strain curve for a member is significant because at this point a rapidly changing relative stress distribution starts in the member and ultimate failure soon follows. This abrupt redistribution of stress may be caused by having passed the critical buckling load in one or more plates in the member

Table V. Relationship Between the Test Ultimate Buckling Loads and the Proportional Limit of the Stress-Strain Curves for All Members

Member	b/t	Test σ_{ult} psi	Proportional Limit of Stress-Strain Curve for Member psi	Proportional Limit ÷ Test σ_{ult} per cent	Type of Failure
B-4	72.5/43.5	29,000	26,000	89.6	Elastic
B-5	72.7/43.9	28,000	27,000	96.4	Buckling
D-1	80.1	19,350	17,000	87.2	
D-2	73.7	25,100	21,000	83.7	
D-3	76.0	24,200	21,000	86.7	
A-1	42.3	32,700	27,500	84.0	Inelastic
A-2	40.4	31,600	22,500	71.1	Buckling
A-3	39.7	38,300	24,500	77.0	
A-4	39.0	34,800	26,500	76.2	
B-1	41.8/33.6	33,600	28,500	84.9	
B-2	40.5/20.3	35,900	28,000	78.0	
B-3	50.3/30.3	28,000	21,000	75.0	
C-1	37.4	--	26,500	--	
C-2	37.8/22.8	34,000	26,000	76.5	
C-3	39.8	29,100	25,000	86.0	
D-5	50.5	27,600	21,000	76.0	
D-6	32.4	39,300	27,000	68.9	
D-7	29.6	46,800	32,500	69.5	

(elastic buckling) or by having reached the yield point at one or more corners of the member (inelastic buckling). There is a reasonably close relationship between the proportional limit of the stress-strain curve of the member and the ultimate strength of the member. For the 12 members which were loaded to the ultimate and which failed by inelastic buckling, the proportional limit averaged 77 per cent of the ultimate load with all members within a range of +12 per cent to -11 per cent of the average. For the five members which failed by elastic buckling, this average was 89 per cent with all members within a range of +9 per cent to -6 per cent of the average. The difference between the averages for the two types of action indicates that when the buckling load of the plates in a member is reached by elastic buckling action, ultimate failure is brought about more rapidly than if the member fails inelastically by allowing a localized yielding of the material at the corners to build up to ultimate failure. The average proportional limit of all members within a range of b/t ratios from 32 to 42 fabricated from $\frac{1}{4}$-inch plates was 27,400 psi with all such members within a range of +4 per cent to -5 per cent of the average. The member D-7 was excluded since it was of such stocky proportions as to act as a compression block. See Table VI. These members failed by inelastic buckling. This comparatively narrow range of values shows a degree of consistency in the physical aspects

Table VI. Proportional Limits of the Stress-Strain Curves for the Members Fabricated from ¼-Inch Plates

Member	b/t	Proportional Limit of Stress-Strain Curve for Member psi
A-1	42.3	27,500
B-1	41.8/33.6	28,500
B-2	40.5/20.3	28,000
C-1	37.4	--
C-2	37.8/22.8	26,000
D-6	32.4	27,000

of the members and of the laboratory procedure.

A consistent relationship between E and E_m over the entire range of b/t ratios used in this investigation was not apparent. See Table VII. Assuming E of the material to be the same in both tension and compression, the E_m of a member which begins an appreciable buckling when the load is first applied and finally fails by elastic buckling should be lower than E. The potential stiffness of all the material in the member is not utilized. Such an effect showed up in member D-1. For a member which does not buckle appreciably until inelastic action occurs, the reduction in E should be negligible since the entire cross section is almost fully effective. Such a relationship was not consistently shown in the test results. However, a consistent relationship between E and E_m was shown for all members fabricated from $\frac{1}{4}$-inch plates. For such members, E_m averaged 98.1 per cent of E with all members within a range of +2 per cent to $-1\frac{1}{2}$ per cent of the average. See Table VIII.

C. Comparison of Test and Calculated Ultimate Buckling Loads

The test ultimate loads of the members of square cross section are listed with the ultimate loads computed by Timoshenko's large deflection theory (48) and by Von Karman's equivalent-width method (53) in Tables IX and X. Since both these methods assume hinged-edge support, a comparison between

Table VII. Relationship Between E of the Material and E_m of the Member

Member	b/t	Tensile E psi x 10^6	Member E_m psi x 10^6	$E_m \div E$ per cent	Type of Failure
B-4	72.5/43.5	30.3	29.9	98.7	Elastic
B-5	72.7/43.9	30.3	30.5	100.5	Buckling
D-1	80.1	30.3	27.6	91.1	
D-2	73.7	30.3	29.5	97.5	
D-3	76.0	30.3	30.3	100.0	
A-1	42.3	29.9	29.8	99.6	Inelastic
A-2	40.4	31.4	29.4	93.9	Buckling
A-3	39.7	30.3	28.9	95.4	
A-4	39.0	31.3	28.9	92.4	
B-1	41.8/33.6	29.9	29.1	97.5	
B-2	40.5/20.3	29.9	30.0	100.1	
B-3	50.5/30.3	31.4	30.3	96.8	
C-1	37.4	29.9	29.1	97.4	
C-2	37.8/22.8	29.9	29.2	98.0	
C-3	39.8	30.3	29.9	98.7	
D-5	50.5	31.4	29.0	92.4	
D-6	32.4	29.9	29.0	97.0	
D-7	24.6	29.9	28.9	96.6	

Table VIII. Relationship Between E of the Material and E_m of the Member for All Members Fabricated from $\frac{1}{4}$-Inch Plates

Member	b/t	Tensile E psi x 10^6	Member E_m psi x 10^6	$E_m \div E$ per cent	Type of Failure
A-1	42.3	29.9	29.8	99.6	Inelastic Buckling
B-1	41.8/33.6	29.9	29.1	97.5	
B-2	40.5/20.3	29.9	30.0	100.1	
C-1	37.4	29.9	29.1	97.4	
C-2	37.8/22.8	29.9	29.2	98.0	
D-6	32.4	29.9	29.0	97.0	
D-7	24.6	29.9	28.7	96.6	

the theoretically-computed strengths and test strengths was made only for members of square cross section.

The values of E and σ_{yp} from the tensile coupon tests were used in computing the ultimate strengths by the large deflection theory.

For the members which failed by elastic buckling, the ultimate buckling loads predicted by the large deflection theory were in satisfactory agreement with the test ultimate buckling loads except for member D-1. For the members which failed by inelastic buckling, the large deflection theory gave values which were much too high and which would be extremely unsafe to use. However, it should be noted that this theory is based upon energy considerations in a plate with an assumed buckled shape. The difference between test and theoretical values may be caused by the difference between the assumed and the actual buckled shapes of the plates. This difference was most pronounced in the members which failed inelastically. For these members, failure was predominantly caused by yielding of the material.

The Von Karman equivalent-width method gave results which were in general unsatisfactory when the theoretically-derived value of C = 1.9 was used. See Table X. For the members which failed elastically, the theoretical values of ultimate buckling loads were about 10 per cent too high for members D-2 and D-3 and about 25 per cent too high for member D-1. For the members which failed inelastically, all theoretical values of

Table IX. Comparison of Ultimate Buckling Loads by Test and by the Timoshenko Large Deflection Theory*

Member	b/t	Test Ultimate Strength psi	Theoretical Ultimate Strength psi	Type of Failure
D-1	80.1	19,350	24,000	Elastic
D-2	73.7	25,100	25,700	
D-3	76.0	24,200	25,050	
A-1	42.3	32,700	53,000	Inelastic
A-2	40.4	31,600	51,040	
A-3	39.7	38,300	53,500	
A-4	39.0	34,800	61,200	
C-1	37.4	--	62,650	
C-3	39.8	29,100	53,300	
D-4	64.4	24,500	28,200	
D-5	50.5	27,600	37,340	
D-6	32.4	39,300	76,900	
D-7	24.6	46,800	119,200	

*Timoshenko, S. Theory of Elastic Stability. 1st ed. New York, McGraw-Hill. 1936. p. 395.

Table X. Comparison of Ultimate Buckling Loads by Test and by Von Karman Equivalent-Width Method

Member	Test Ultimate Strength psi	Equivalent-Width Method				Type of Failure
		C^1	Ultimate Strength psi	C^2	Ultimate Strength psi	
D-1	19,350	1.9	24,700	1.50	19,500	Elastic
D-2	25,100	1.9	27,250	1.40	20,100	
D-3	24,200	1.9	26,800	1.40	19,700	
A-1	32,700	1.9	54,800	1.20	34,600	Inelastic
A-2	31,600	1.9	47,500	0.95	23,800	
A-3	38,300	1.9	49,400	1.00	26,000	
A-4	34,800	1.9	57,700	1.10	33,400	
C-1	--	1.9	61,600	1.10	35,700	
C-3	29,100	1.9	49,000	1.00	25,800	
D-4	24,500	1.9	29,700	1.30	20,300	
D-5	27,600	1.9	38,300	1.20	24,200	
D-6	39,300	1.9	73,000	0.95	36,500	
D-7	46,800	1.9	100,100	0.80	42,200	

[1] C = 1.9 from theoretical derivation.

[2] C most nearly fitting test results of very thin plates as determined by E. E. Sechler.

Timoshenko, S. Theory of Elastic Stability. 1st ed. New York, McGraw-Hill. 1936. p. 402.

ultimate buckling loads were much too high and are unsafe to use.

However, a fairly satisfactory agreement between test and predicted values by the equivalent-width method was obtained when the Sechler values for C were used. The predicted values were safely below the test values for all members except member A-1 and member D-1. The predicted ultimate buckling load was about 6 per cent higher than the test value for member A-1 and less than 1 per cent too high for member D-1. For the members which failed elastically, the predicted values of ultimate buckling loads were too low by about 20 per cent of the test values for members D-2 and D-3. The values agreed almost perfectly for member D-1. For the members which failed inelastically, the predicted value for A-1 was about 6 per cent too high, for members A-2 and A-3 about 25 per cent too low, and for the remainder of the members about 7 to 16 per cent too low.

D. Buckling Patterns and General Behavior of the Plates in the Test Members

A study of the load-deflection profiles shows that the wave patterns of the actual buckled plates were entirely different from those predicted theoretically in idealized plates. See Appendix A. The buckled shape of the plate under loading was largely dependent upon the initial irregularity

in the plate. However, in all cases, at any given cross section of the member where the maximum buckling occurred, two opposite sides always buckled inward and the remaining two sides outward as predicted from theoretical considerations.

In all members which failed by elastic buckling, the plates assumed a pattern of buckling governed by the initial shapes of the plates with the exception that in certain cases, a crest or dent in a plate would reverse under loading in order to accommodate the shape taken by an adjacent plate which, due to its initial shape, had a greater effect on the final shape of the member as a whole. Appreciable deflection at critical points along the centerline of each plate began immediately upon application of the load. The deflections grew slowly at first but increased rapidly as the point of instability was approached. Load-deflection curves for the critical points on two adjacent plates of a member gave critical buckling loads of about the same magnitude. Since any two plates in a member would rarely have exactly the same critical buckling strength, it appears that when one plate becomes unstable it no longer gives the required edge support to the adjacent plates and instability of all plates in the member soon follows. This would apply particularly to square cross sections with walls of equal thickness. During the laboratory tests of the members with square cross sections, visible buckling appeared simultaneously in all plates in the member. The strength of the weakest plate in the member

appeared to govern closely the strength of the member.

For most members which failed inelastically, no measurable load deflections occurred until sizeable loads were reached (10,000 to 15,000 psi). Deflections did not become appreciable until after the yield point had been reached at one or more corners of the member. The tendency for the buckled shape to follow the general shape of the initial irregularities was not as pronounced as was the case for the members which failed elastically. It is suggested that the corner of the member first reaching the yield point of the material initiated buckling and strongly influenced the final buckled shape of the member. However, the initial irregularities in the plates would influence the distribution of the load and thus indirectly influence the manner of buckling.

The side with the longitudinal welded joint did not appear to initiate buckling in any member although this side was usually the most deformed in the process of fabrication. The welded bead acting as a longitudinal stiffener on the interior of the member may have given additional strength to the side.

A definite correlation between the degree of initial irregularity in the plates of the members and the buckling strengths of the plates was not possible from the results of these tests. The general behavior and buckling strength of each plate in a member is affected by the behavior and buckling

Fig. 15. Typical Secondary Buckling Failure of Test Members.

strength of the other three plates in the member. There are an infinite number of possible initial shapes, and combinations thereof, which may exist in the four plates of a member, even though the maximum amplitude of the initial waves in the plates may be of the same order of magnitude. However, members B-4 and B-5 were of almost identical nominal dimensions; the initial irregularities were of the same order of magnitude but were shaped differently. Yet, these members behaved substantially the same. A similar comparison may be made of members D-2 and D-3. Hence, it is felt that the behavior of the test members was highly indicative of what can be expected from members of this type with similar degrees of initial irregularity. Figure 15 shows typical buckling failures.

E. Secondary Buckling and Model Analysis

Definite conclusions as to the dependability of model analysis in predicting the secondary buckling characteristics of plates under laboratory conditions could not be made from the results of this investigation. However, conclusions of an indicative nature were made by (1) observing the geometrical properties of the members used in this investigation, (2) observing the requirements of similitude, and (3) observing the conclusions of other investigators.

It is interesting to observe the similitude requirements necessary for a true model analysis involving elastic secondary

buckling. The requirements necessary for determining the critical buckling stress from a model may be determined as follows:

Item No.	Symbol	Factors Affecting Critical Buckling Stress	Basic Dimension
1	σ_{cr}	Critical buckling stress	FL^{-2}
2	l	Length	L
3	b	Long width	L
4	d	Short width	L
5	t	Thickness	L
6	E	Young's Modulus	FL^{-2}
7	μ	Poisson's Ratio	-
8	S	Shape Factor	-

By the Buckingham Pi Theorem (33, p. 36) the number of quantities involved minus the number of basic dimensions involved gives the number of dimensionless and independent Pi terms which may be used to define the relationship. In this case six Pi terms are required and the relationship may be expressed as,

$$\frac{\sigma_{cr}}{E} = f\left(\frac{b}{l}, \frac{d}{l}, \frac{t}{l}, \mu, S\right)$$

Since this relationship applies to both model and prototype, the design and prediction equations may be set up as follows:

Prediction equation $\frac{\sigma_{cr}}{E} = \left(\frac{\sigma_{cr}}{E}\right)_{mod}$

or $\sigma_{cr} = E\left(\frac{\sigma_{cr}}{E}\right)_{mod}$

Design conditions

(1) $(b/l)_{mod} = b/l$

(2) $(d/l)_{mod} = d/l$

(3) $(t/l)_{mod} = t/l$

(4) $(\mu)_{mod} = \mu$

(5) $(S)_{mod} = S$

(6) Boundary conditions the same.

Design conditions 1, 2, and 3 demand that the dimensions involved be geometrically similar for the model and prototype.

Design condition 4 demands that Poisson's Ratio be the same for the model and prototype.

Design condition 5 requires that the effect of the initial irregularities in the plates in the model be the same as the effect of such irregularities on the behavior of the prototype. This shape-factor condition will rarely be exactly satisfied; the question then becomes one as to how serious the non-fulfillment of this design condition may be.

Design condition 6 requires that the manner of loading and degree of edge support be the same for both model and prototype.

Design condition 5 is probably the most difficult condition to reasonably satisfy. Wyly (59, p. 73) has observed that:

> . . . The effect of initial kinks or short waves in the plate will be to lower the buckling strength. . . . The thinner the material the greater is the probable effect of such kinks. Probability of initial kinks increases with decreasing thickness.

Magnitude of kink compared with thickness of plate becomes greater with decreasing thickness.

In Table XI is shown the ratio of the maximum measured amplitude of the initial irregularities in the plates to the thickness of the plates. There is a general trend for this ratio to become larger with decreasing plate thicknesses.

The members shown in Table XI failed by inelastic buckling, hence a study by model analysis insofar as critical buckling loads were concerned could not be made. When the investigation was planned, there was no reason to anticipate whether the members would fail by elastic or inelastic buckling. However, an attempt was made to observe the degree of similarity of behavior of these members while acting within the elastic limit of the material. The principles of similitude were used to predict the total load on the prototype which would produce the same average unit strain at the four corners of the prototype as was produced in the model by an observed load on the model. The design and prediction equations were set up as follows:

Item No.	Symbol	Principal Factors Affecting Longitudinal Unit Strain	Basic Dimensions
1	ϵ	Unit Strain	-
2	P	Total load on the member	F
3	A	Total area of member	L^2
4	E	Young's Modulus	FL^{-2}
5	S	Shape factor	-
6	μ	Poisson's Ratio	-

Table XI. Ratio of Amplitude of Initial Irregularities to Plate Thickness for Members Nearly Geometrically Similar

Member	b inches	t inches	b/t	Maximum Measured Amplitude of Initial Irregularity inches	Maximum Amplitude ÷ Plate Thickness per cent
A-4	3.0	0.075	39	0.06	80
A-3	5.2	0.131	40	0.03	23
A-2	8.2	0.203	40	0.02	10
A-1	10.1	0.238	42	0.05	21
C-1	10.1	0.270	37	0.05	19

Using the Buckingham Pi Theorem, the relationship may be set up as:

$$\frac{P}{AE} = f\ (\epsilon\ ,\ S,\ \mu\ )$$

The design and prediction equations may be set up as:

Prediction equation:

Design conditions:

(1) $\epsilon_{mod} = \epsilon$

(2) $\mu_{mod} = \mu$

(3) $S_{mod} = S$

Design conditions 1 and 2 were substantially satisfied by the members of Table XI. However, design condition 3 which includes the effect of the proportions of the cross sections of the members as well as the initial irregularities in the plates of the members may not be reasonably satisfied. The purpose of this analysis is to indicate the variation caused by the degree of non-fulfillment of condition 3 which is present in those members.

The members of the A series were made as nearly geometrically similar as was practicable under usual shop conditions; the b/t ratios varied from 37 to 42. Member C-1 had a cross section closely similar to the A series but had a greater length. However, this member was included since length does not appear to influence materially the factors governing

inelastic buckling.

In Table XII-A, the member A-4 was used as the model and the remainder of the members were used as prototypes. The largest prototype had an "n" value of about 3½ times the size of the model. An average unit strain of 500 micro-in per in (corresponding to about 15,000 psi) was used as the base value of strain for comparing predicted and actual loads. The variation between predicted and actual values of loads varied from about 3 to 9 per cent of the actual loads.

In Table XII-B the member A-3 was used as the model. Here the largest prototype had an "n" value of about two times the size of the model. The variation between predicted and actual values of loads varied from about 1 to 6 per cent of the actual loads.

The results indicate that better predicted loads were obtained when member A-3 was used as a model with a maximum length scale factor of about 2 than when A-4 was used as a model with maximum length scale factor of about 3½. This trend in the value of "n" seems to indicate that the larger the length scale factor, the larger the deviation in model predictability caused by the increasing amplitude-thickness ratio for decreasing thickness of plates. Tests by other investigators have shown that a length scale factor of 2 may be satisfactorily used for plates in built-up sections (17). The results of this study indicate that a length scale

Table XII-A. Model Analysis of Geometrically-Similar Test Members Using Member A-4 as a Model

Member	Model	Prototypes			
	A-4	A-3	A-2	A-1	C-1
b/t	39	40	40	42	37
Area sq. in.	0.89	2.64	6.44	9.05	10.3
n	1	1.74	2.73	3.37	3.37
Load P (Predicted) lbs	-	36,300	91,400	123,000	149,500
Load P* (Actual) lbs	12,620	38,000	95,000	135,000	144,000
Error per cent of Actual P	-	4.5	3.8	8.9	2.5

*Unit stresses for ϵ = 500 micro inches per inch.

Table XII-B. Model Analysis of Geometrically-Similar Test Members Using Member A-3 as a Model

Member	Model	Prototypes			
	A-3	A-4	A-2	A-1	C-1
b/t	40	39	40	42	37
Area sq. in.	2.64	0.89	6.44	9.05	10.3
n	1	0.58	1.58	1.95	1.95
Load P (Predicted) lbs	-	13,350	96,000	128,600	146,000
Load P* (Actual) lbs	38,000	12,620	95,000	135,000	144,000
Error per cent of actual P	-	5.8	1.1	4.7	1.4

*Unit stresses for ϵ = 500 micro inches per inch.

factor of 2 may also be satisfactorily used for members of the type investigated in this study.

V. SUMMARY AND CONCLUSIONS

A study was made of secondary buckling in hollow rectangular compression members, each of which was fabricated from a single mild structural steel plate with a longitudinal welded joint along one side of the member. The thickness of the material used varied from 0.075 to 0.270 inches. The b/t ratios varied from 24.5 to 80.1 with both square cross sections and rectangular cross sections used; a b/d ratio of 0.6 was used for the rectangular sections.

The objectives of this study were: (1) To determine the general structural behavior of this type of column section under axial loads insofar as secondary buckling was concerned; (2) To indicate a safe and reliable design procedure; (3) To indicate the probable degree of dependability with which model analysis may be used if both prototype and model are fabricated under ordinary shop methods.

To determine the initial shape of the unloaded plates and the load deflection of the plates under various applied loads, the plate contour meter was built. The critical buckling loads for plates of members which failed by elastic buckling were found by plotting load-deflection curves for critical points determined from load-deflection profiles of the sides of the members. Readings were taken on two adjacent

sides of each member, one of which was usually the side most visibly deformed. Two SR-4 electrical-resistance strain gages were placed at each corner at midheight of each member for determining the load distribution in the member.

Test critical buckling loads were compared with the theoretical Bryan loads. Curves representing the buckling resistance of the test members were proposed. Working curves proposed by other investigators were compared with the working curve proposed as a result of this investigation. A study was made of the relationship between the average measured unit strain at the corners of the member and the average applied load per unit area. A comparison was made of the test ultimate strengths and the ultimate strengths predicted by Timoshenko's large deflection theory and by Von Karman's equivalent-width method. A study was also made of the load-deflection profiles and plate buckling patterns. Certain aspects of model analysis were also briefly discussed.

The following is a summary of the more important results and observations obtained in this investigation and the conclusions drawn therefrom:

(1) The secondary buckling strengths of the plate elements of the members tested were dependent on the width-thickness ratios of the plates, the degree of edge support along the unloaded edges of the plates, the properties of the materials, the initial irregularities in the plates and the

degree of departure from the ordinarily assumed idealized testing conditions.

(2) The column sections tested exhibited favorable secondary buckling characteristics for sections with rounded instead of ideal square corners.

(3) The plate contour meter very satisfactorily fulfilled its intended functions of determining the shape of the unloaded plates and the deflection changes in the plates under various increments of loading.

(4) Elastic buckling failure for members of square cross section occurred in the b/t ratio range of 70 to 80 and probably occurred in the 60 to 70 range also.

(5) Inelastic buckling failure for members of square cross section occurred in the b/t ratio range of 24 to 50 and probably in the 50 to 60 range also.

(6) The Bryan formula with k = 4.0 may be used in the b/t ratio range of 70 to 80 for square cross sections if the overall width of the plate is used to compute the b/t ratio. For a b/t ratio less than 70, the proposed test curve for buckling resistance of the members should be used. The equation of this curve is $\sigma = 5\ b^2/t^2 - 1080\ b/t + 71{,}000$ and is valid for a b/t of 20 to 80.

(7) For members of rectangular cross section with a b/d ratio of 0.6, the Bryan values were not reached in the range of b/t ratios tested. The proposed curve for buckling

resistance of the test members should be used. The equation of this curve is $\sigma = 6.1\ b^2/t^2 - 1025\ b/t + 65{,}000$ and is valid for a b/t of 40 to 80.

(8) Below a b/t ratio of about 50, the members of square cross section and rectangular cross section behaved in substantially the same way as indicated by the proposed curves for buckling resistance of the test members. Above a b/t ratio of about 50, the proposed values for buckling resistance of the rectangular sections are higher than those for the members of square cross section.

(9) The effects on the buckling strength of the plates caused by the initial irregularities, the increased stability offered by the rounded corners of the members, and the degree of edge support offered by this type of construction could not be segregated but the combined effects were obtained.

(10) There was a general trend of agreement between the curve proposed by George E. Beggs for the buckling resistance of structural steel plates in square cellular members and the curve which best fitted the results of this investigation for members of square cross section.

(11) To arrive at allowable design plate loads for the members used in this investigation, it was considered more justifiable and consistent to apply a reasonable safety factor to the test curve representing the buckling resistance of the members than to apply a safety factor to the Bryan curve.

(12) The maximum measured displacements at the unloaded edges of the plates of the members which failed elastically were less than 1/100 inch when the critical buckling loads were reached.

(13) The maximum measured load deflections at the critical buckling loads were about one-third the plate thickness for members of square cross section and about one-half the plate thickness for the members of rectangular cross section (b/d = 0.6) which failed elastically.

(14) Members C-1, C-2, and C-3 behaved in substantially the same way as the corresponding members A-1, B-2, and A-3 which had cross sections of similar proportions but were of different lengths.

(15) The equality of load distribution obtained in the laboratory was sufficient for reliable results. Members B-4 and B-5 were of almost identical proportions and test results from the two members checked favorably. This was also true of members D-2 and D-3.

(16) The stress-strain curve for a member was very helpful in evaluating the behavior of the member. The proportional limit of the curve indicated the average unit load at which the yield point was reached at one or more corners of the member or at which the critical buckling load had been passed.

(17) For all members which failed inelastically, (b/t ratios from 24 to about 60) the proportional limits of the

stress-strain curves for the members averaged 77 per cent of the ultimate loads for the members with a range of about ± 11 per cent of the average.

(18) For all members which failed elastically, (b/t ratios from about 60 to 80) the proportional limits of the stress-strain curves for the members averaged 89 per cent of the ultimate loads with a range of about ± 7½ per cent of the average.

(19) The proportional limits of the five members tested to ultimate failure and which were fabricated from ¼-inch plates (b/t ratio from 32 to 92) averaged 27,400 psi with a range of ± 4½ per cent of the average.

(20) A consistent correlation between E and E_m over the entire range of b/t ratios tested was not found.

(21) For the seven test members fabricated from ¼-inch plates, E_m averaged 98.1 per cent of E with a range of about ± 1½ per cent of the average.

(22) The computed theoretical ultimate buckling loads of the members of square cross section by Timoshenko's large deflection theory were higher than the test values for all members. For the members which failed elastically, the theoretical values were too high by about 3 per cent of the test values for members D-2 and D-3 and too high by about 24 per cent for member D-1. For the members which failed inelastically, the theoretical values were much too high (up to

150 per cent for member D-7) and would be unsafe to use.

(23) The difference between the theoretical ultimate buckling loads by the large deflection theory and the test ultimate buckling loads may be attributed largely to the difference between the theoretically-assumed shape of the buckled plates and the actual shape of the buckled plates. Also, the theoretically-assumed hinged edge conditions are not realized in practice.

(24) The predicted ultimate loads of the test members by the Von Karman equivalent-width method were higher than the test ultimate loads for all members when C = 1.9 was used. For the members which failed elastically, the predicted values were from 10 to 25 per cent higher than the test values. For the members which failed inelastically, the predicted values were from 21 to 110 per cent too high.

(25) Safe predicted values of the ultimate buckling loads were obtained by the equivalent-width method when the experimentally-determined Sechler values of C were used. The predicted values were from 7 to 25 per cent lower than the test values except for members A-1 and D-1 for which the predicted values were up to 6 per cent too high.

(26) The buckled wave patterns of the plates of the test members were entirely different from those predicted theoretically for ideal plates. Application of load on the plates tended to increase the magnitude of the initial irregularities.

(27) For the members which failed elastically, the buckling pattern of the plates was determined almost entirely by the initial irregularities, although occasionally a dent or crest would reverse in order to accommodate the shape assumed by an adjacent plate. For the members which failed inelastically, the correlation between the initial irregularities in the plate and the final buckled shape was not so close.

(28) In the case of elastic buckling failure, the plate deflections grew slowly but increased rapidly as the point of elastic instability was approached.

(29) In the case of inelastic buckling failure, the plate deflection grew very slowly (in some cases were not measurable until a loading of 10,000 to 15,000 psi was reached) and did not become large until occurrence of inelastic action of the material in the member.

(30) The shape and proportions of the cross sections had a significant effect on the behavior of the member. The different rates of buckling of the component plates of a member and the degree of edge support offered each plate by the adjacent plate elements had a substantial effect on the action of the member.

(31) The strength and general behavior of any plate in a member was substantially influenced by the strength and behavior of the other plates in the member. Instability in

one plate of a member accelerated instability in the remaining plates of the member.

(32) The welded joint along one side of each member gave additional buckling strength to that side by acting as a longitudinal stiffener.

(33) Longitudinal strain measurements were not taken on the central portions of the plates because the strains caused by axial loads and those caused by bending of the plates could not be separated.

(34) The fabrication process introduced initial irregularities in the members considerably larger than the initial irregularities in the rolled plates before fabrication.

(35) A definite correlation between the amplitude of the initial irregularities in the plates and the buckling strength of the plates was not possible from the results of this investigation. However, the behavior of the test members was highly indicative of the probable behavior of members of this type of construction with initial irregularities of the same order of magnitude.

(36) Although definite conclusions pertaining to the model analysis of secondary buckling could not be made as a result of this investigation, conclusions of an indicative nature were made by (1) observing the geometrical properties of the test members, (2) observing the theoretical requirements of similitude, (3) observing the conclusions of other

investigators, and (4) studying from a model analysis standpoint the behavior of the A series of test members while acting within the elastic limit of the material. Members of Series A failed by inelastic buckling.

(37) In the test members of Series A, which were nearly geometrically similar, there was a general trend for the ratio of maximum measured amplitude of initial irregularity to plate thickness to become larger with decreasing plate thicknesses.

(38) The results of this investigation indicated that the larger the length scale factor for a model, the greater the deviation in model predictability caused by the increasing amplitude-thickness ratio for decreasing thicknesses of plates; results of this and other investigations indicated that a length scale factor of 2 may be satisfactorily used for predicting secondary buckling characteristics of plates.

VI. RECOMMENDATIONS FOR FURTHER STUDY

One of the results of this investigation has been to indicate several possible extensions of, and additions to, this study. Hence, it is suggested that:

(1) Further tests be made on hollow members of square cross section in the b/t ratio range of 50 to 70 to establish more definitely the transition point above which the members fail by elastic buckling and below which the members fail by inelastic buckling.

(2) Further tests be made on members of rectangular cross section to verify the results of this investigation for members with a 0.6 b/d ratio and to extend this study to rectangular members with various other values for the b/d ratio.

(3) Further tests be made using geometrically-similar members which fail in the elastic buckling range in order to obtain a better correlation between the critical buckling strength of a plate in a member and the ratio of maximum amplitude of initial irregularity in the plate to plate thickness.

(4) Further tests be made involving a sufficient number of members so that probability principles can be applied to take into account the probable irregularity factor which has

been shown to play an important part in the behavior of members in compression.

(5) Tests be made to determine the effect on the critical buckling strength of a member caused by either ovaloid or circular perforations widely spaced along two opposite sides for inspection purposes; also to determine the effect of such perforations closely spaced along the narrower sides of rectangular cross sections.

(6) This study be extended to other geometric combinations such as plates bent into angle and channel sections and plates welded into various proportions of wide flange sections and of H-sections.

(7) Tests be made to determine the feasibility of using stiffening effect of small steel strips welded along the longitudinal centerlines of the plates of a member.

(8) The general scope of this investigation be extended by using eccentric loads instead of axial loads.

(9) A study be made of the flexural behavior of sections made by bending and welding plates using the general proportions and ratios described in the foregoing recommendations.

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VIII. ACKNOWLEDGMENTS

The author wishes to take this opportunity to express his gratitude to Professor Frank Kerekes who suggested this study and who gave much encouragement and valuable assistance in carrying out the project. Thanks are due all other members of the author's committee, Dr. E. W. Anderson, Professor A. H. Fuller, Professor L. S. Linderoth, Dr. Glenn Murphy, Professor L. O. Stewart, and Dr. G. R. Town, all of whom exhibited a friendly interest in the project and all of whom gave much-needed assistance of various kinds. Special thanks are due Dr. G. R. Town, who, as Associate Director of the Engineering Experiment Station, was most cooperative in obtaining much of the equipment, supplies, and personnel necessary for the project. The author's gratitude also goes to Professor L. S. Linderoth who helped conceive the general form of the contour meter, who designed the details, and who gave much assistance in its construction. Professor Henry M. Black, Head of the Department of Mechanical Engineering, was most helpful in arranging for the construction of the contour meter.

The author also wishes to acknowledge Mr. Lloyd Anderson of the Pittsburgh-Des Moines Steel Company. Mr. Anderson displayed a keen and intimate interest in the project and it

was through his efforts that the test members were fabricated and supplied by the Pittsburgh-Des Moines Steel Company.

Mr. John Griffiths of the American Institute of Steel Construction was most helpful in obtaining the steel for the contour meter. In this respect thanks are also due the Pittsburgh-Des Moines Steel Company, the Des Moines Steel Company, and the A. M. Castle Steel Company for supplying the necessary steel for the contour meter.

A P P E N D I C E S

A. Load-Deflection Profiles for All Members

Form No. 1208 — 10 Squares to Inch
AMERICAN PAPER RULER CO., HOLLISTON, MASS.

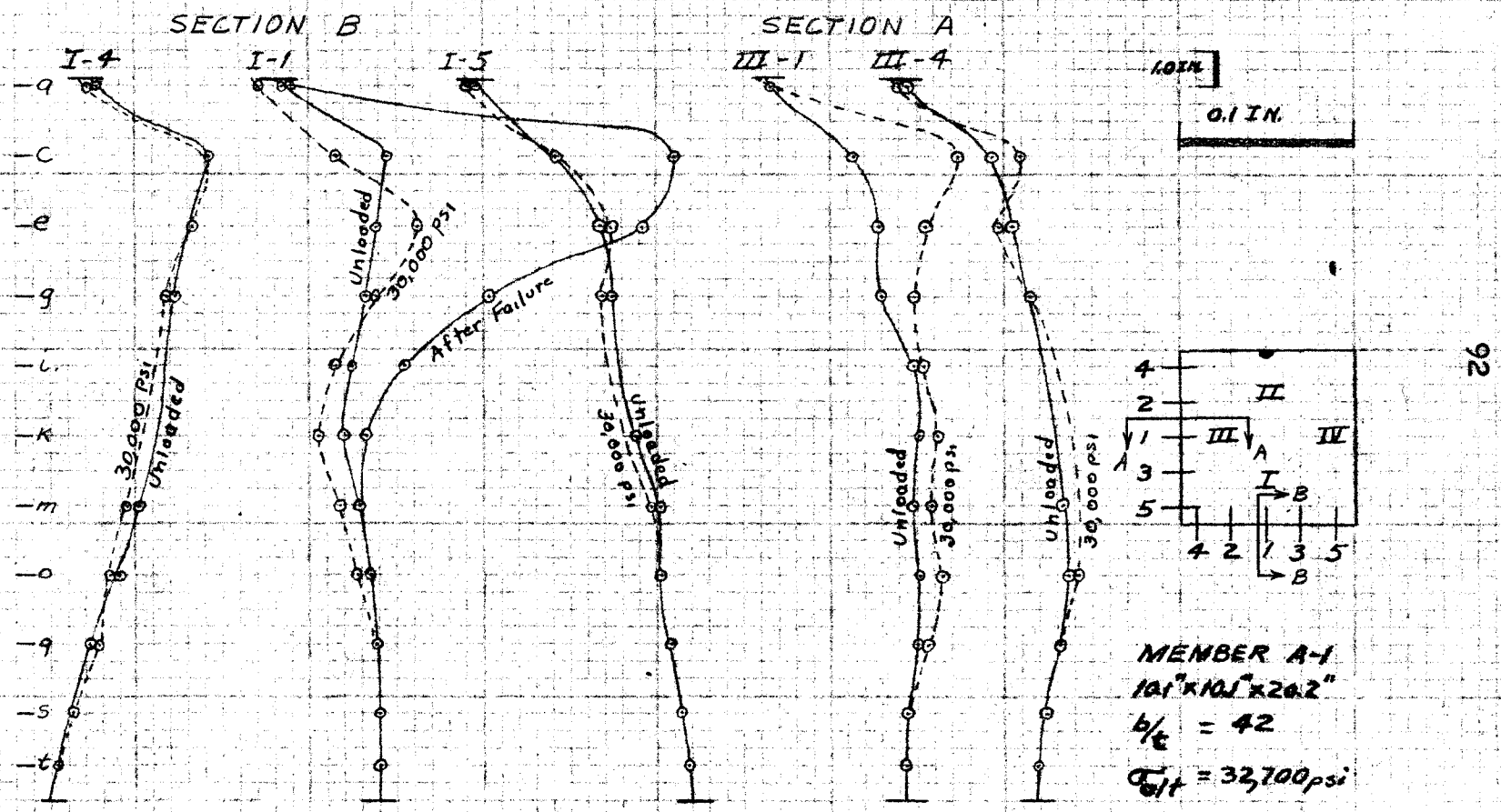

Fig. 16. Load-Deflection Profiles for Member A-1.

Size No. 909 — 10 Squares to Inch
AMERICAN RULED & PAPER CO., HOLYOKE, MASS.

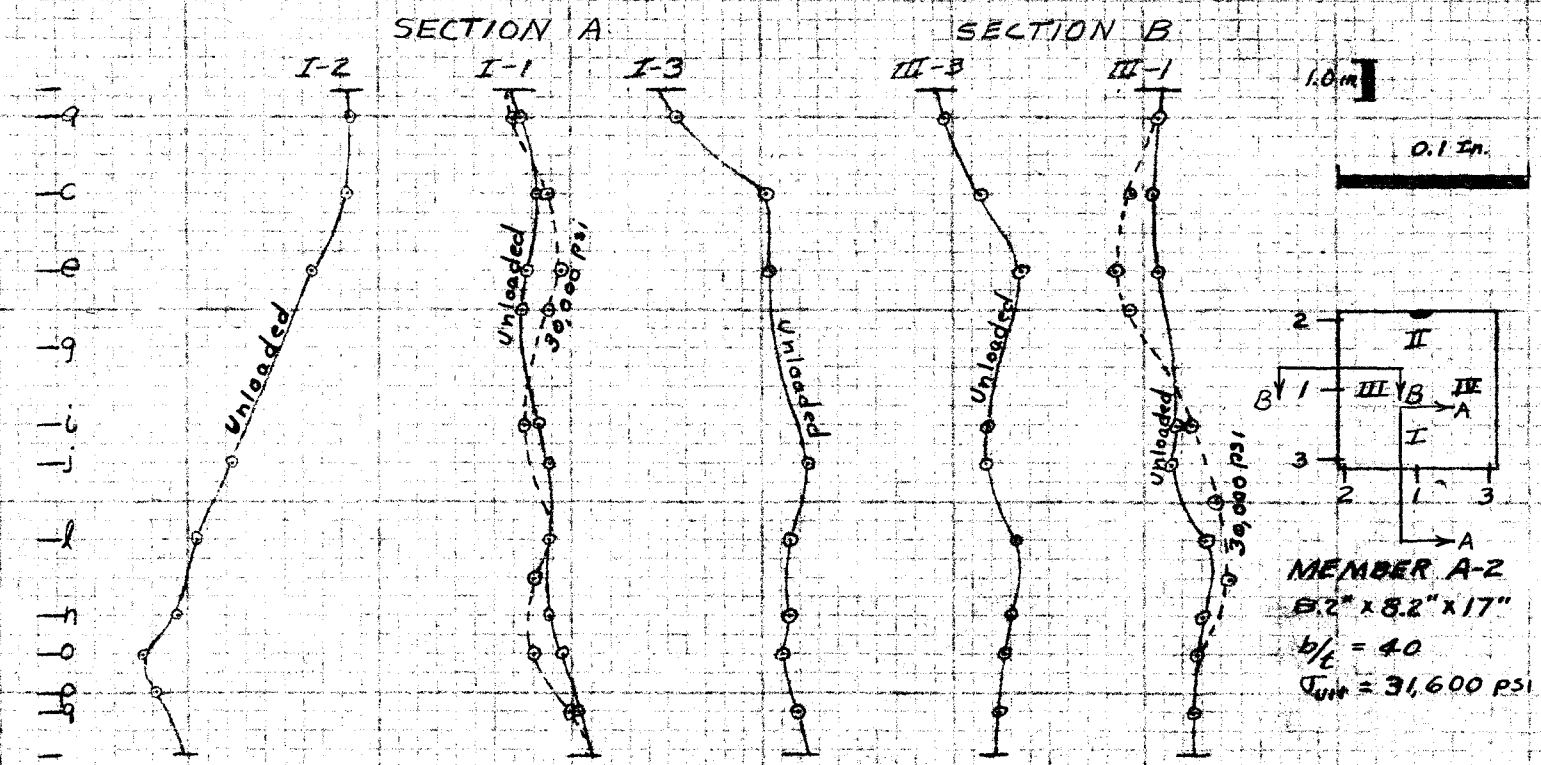

Fig. 17. Load-Deflection Profiles for Member A-2.

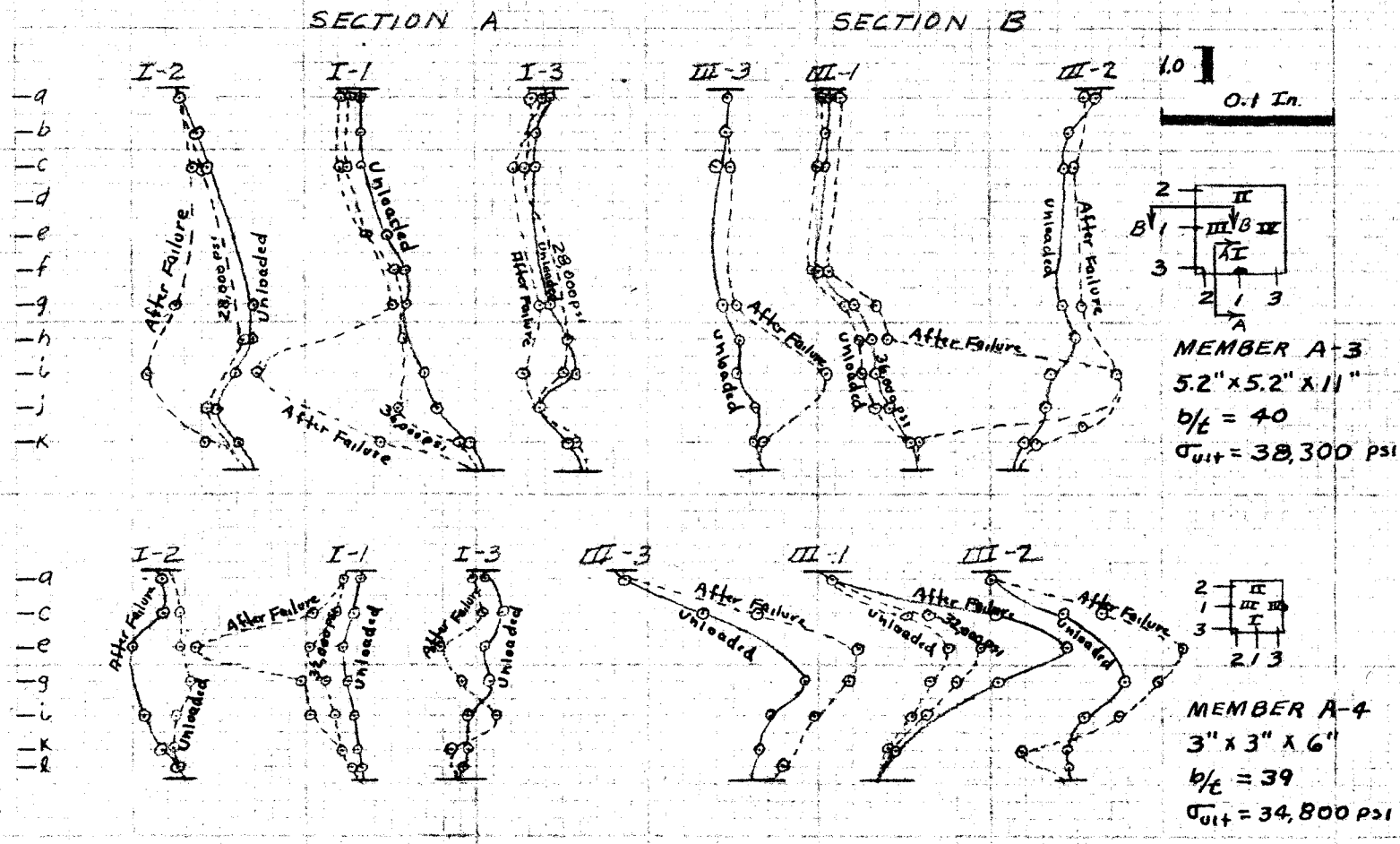

Fig. 18. Load-Deflection Profiles for Members A-3 and A-4.

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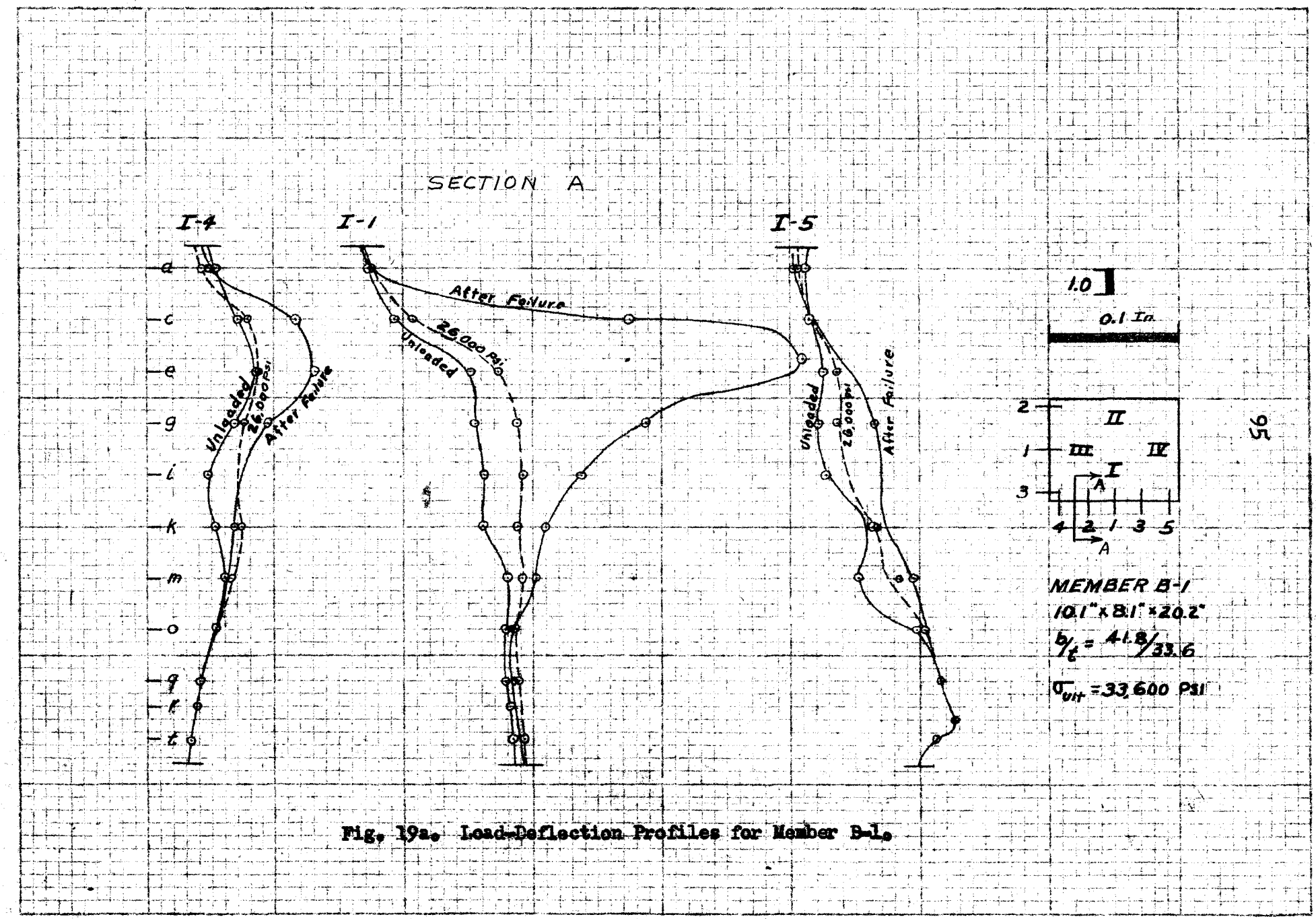

Fig. 19a. Load-Deflection Profiles for Member B-1.

Form No. 858 — 10 Squares to Inch
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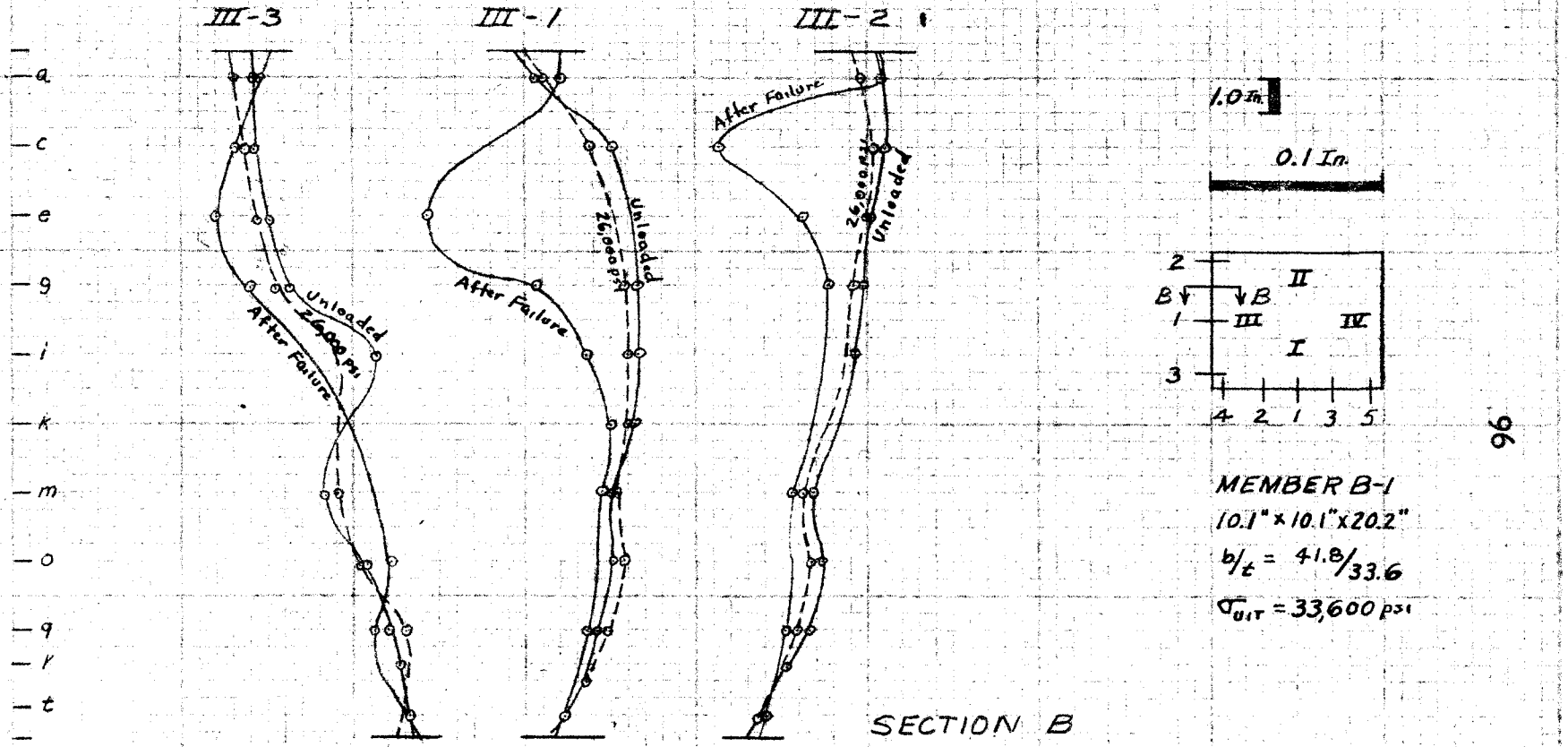


Fig. 19b. Load-Deflection Profiles for Side III, Member B-1.

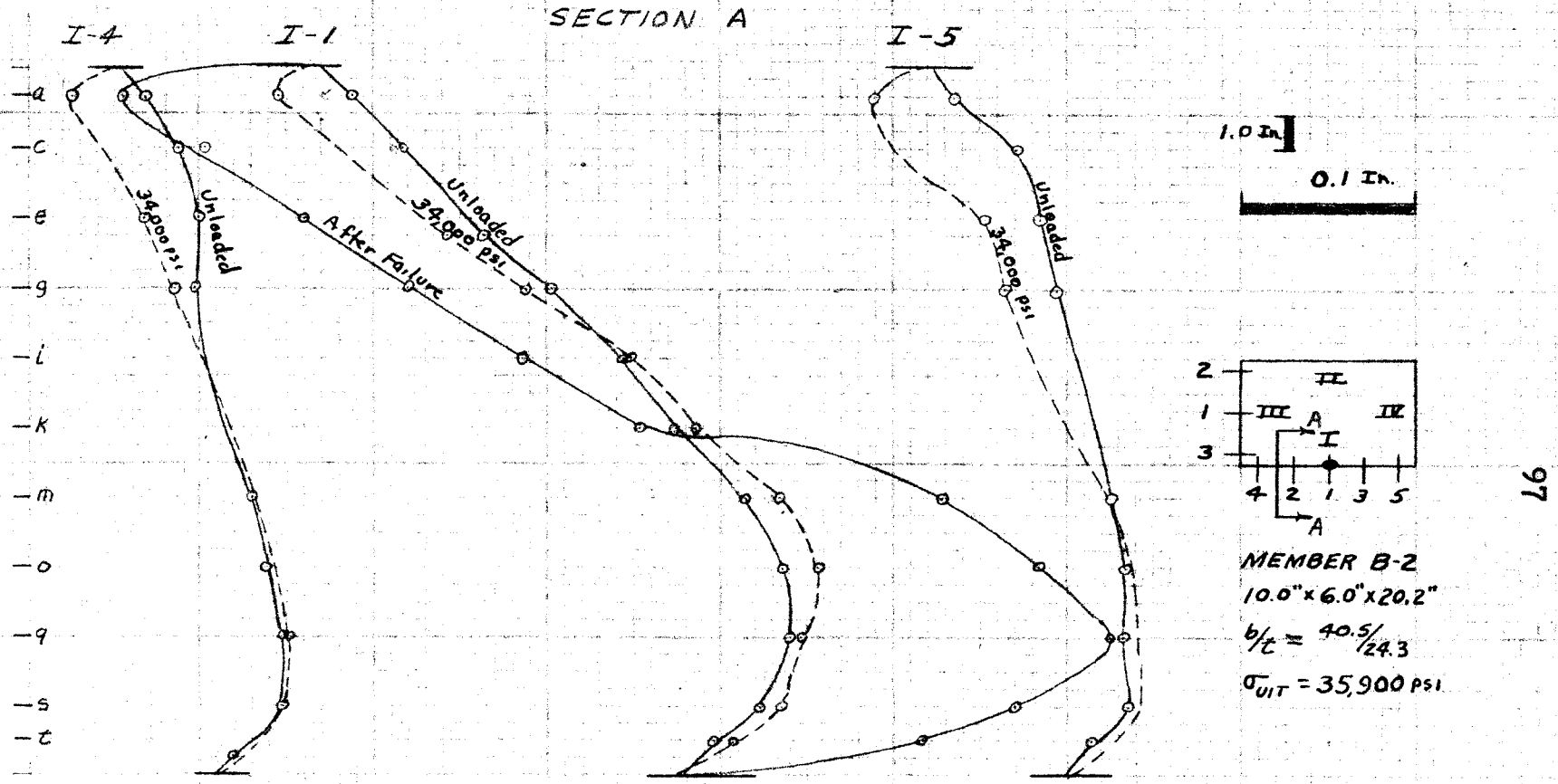


Fig. 20a. Load-Deflection Profiles for Side I, Member B-2.

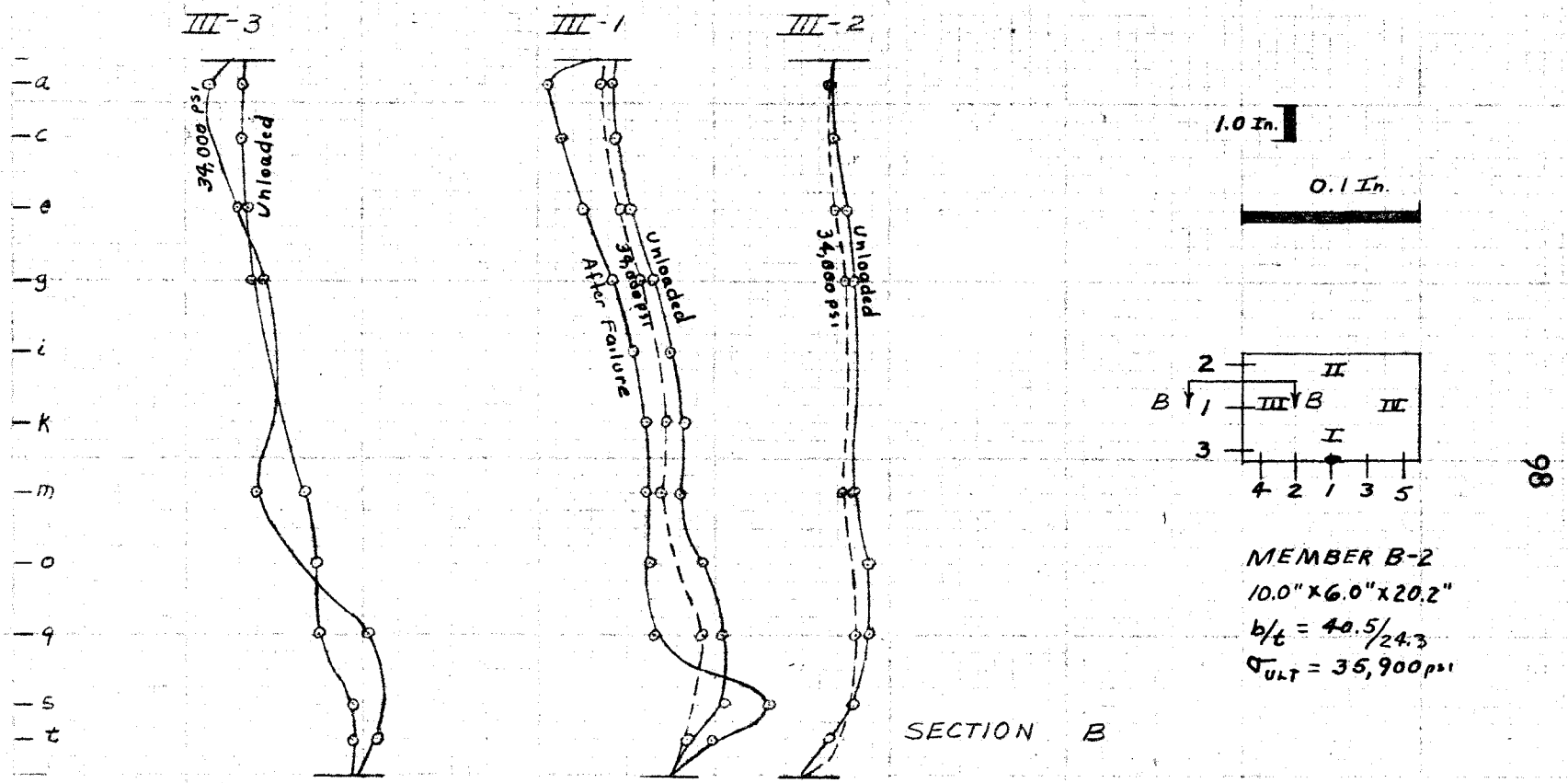


Fig. 20b. Load-Deflection Profiles for Side III, Member B-2.

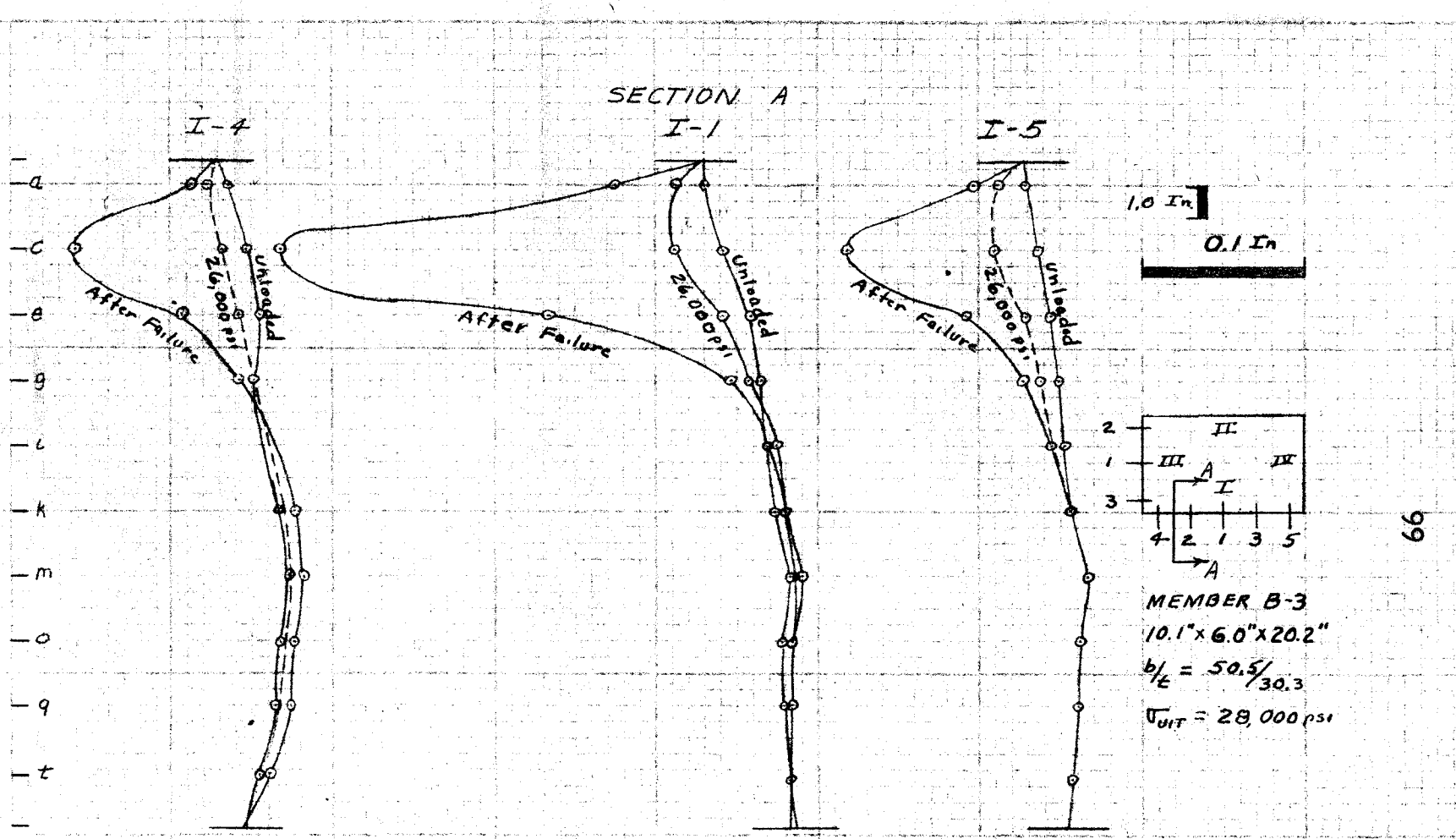


Fig. 21a. Load-Deflection Profiles for Side I, Member B-3.

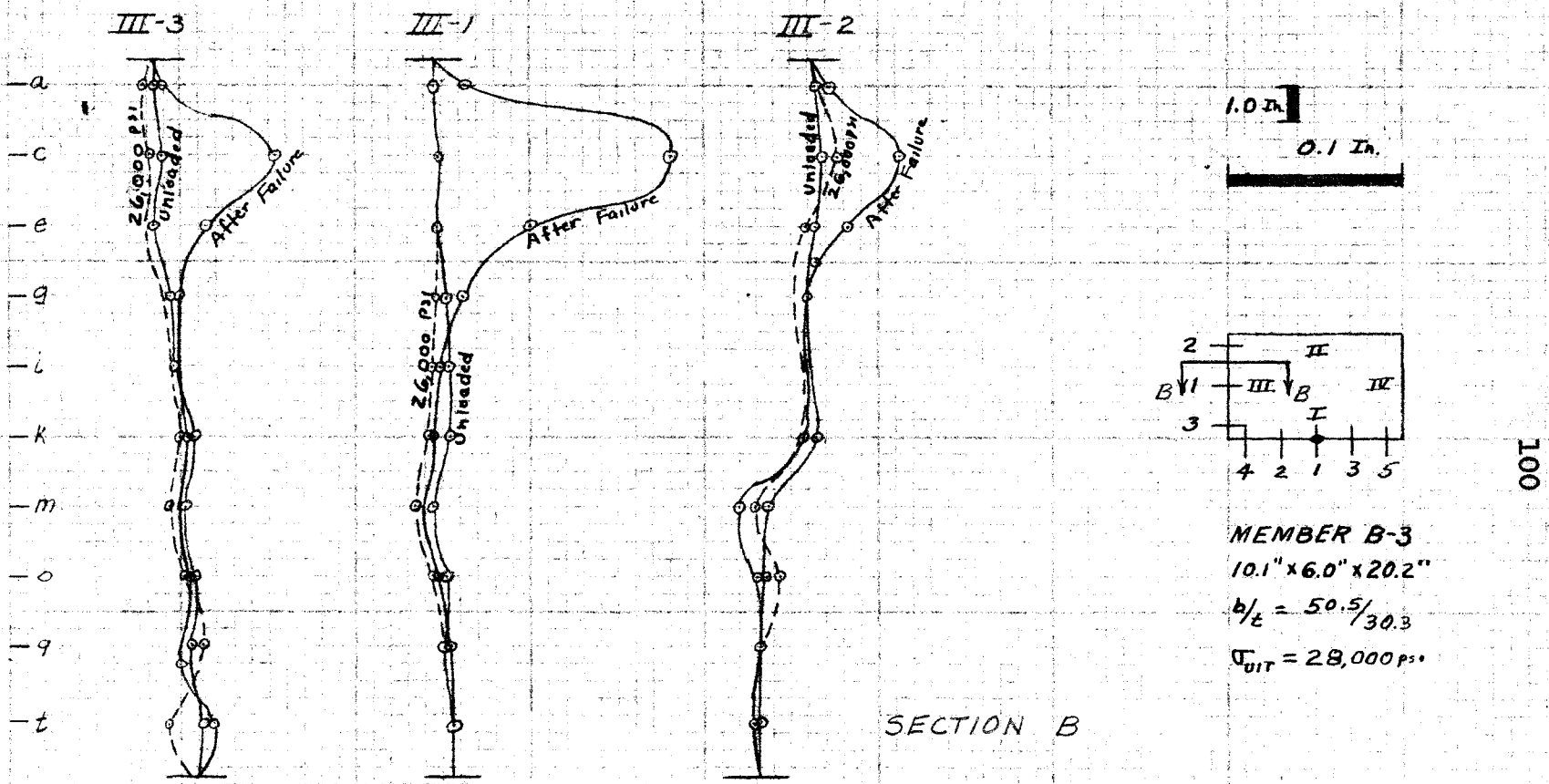


Fig. 21b. Load-Deflection Profiles for Side III, Member B-3.

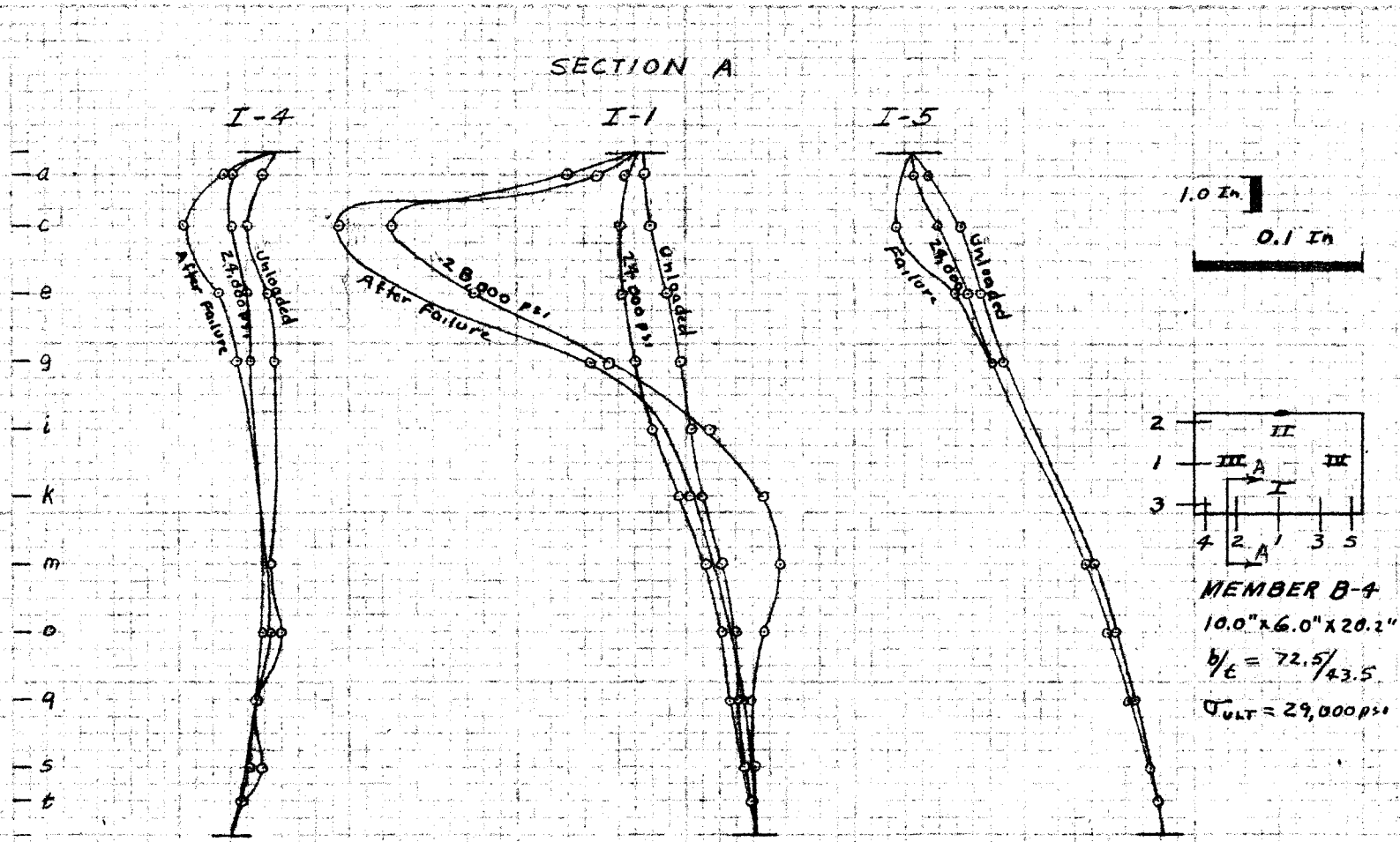


Fig. 22a. Load-Deflection Profiles for Side I, Member B-4.

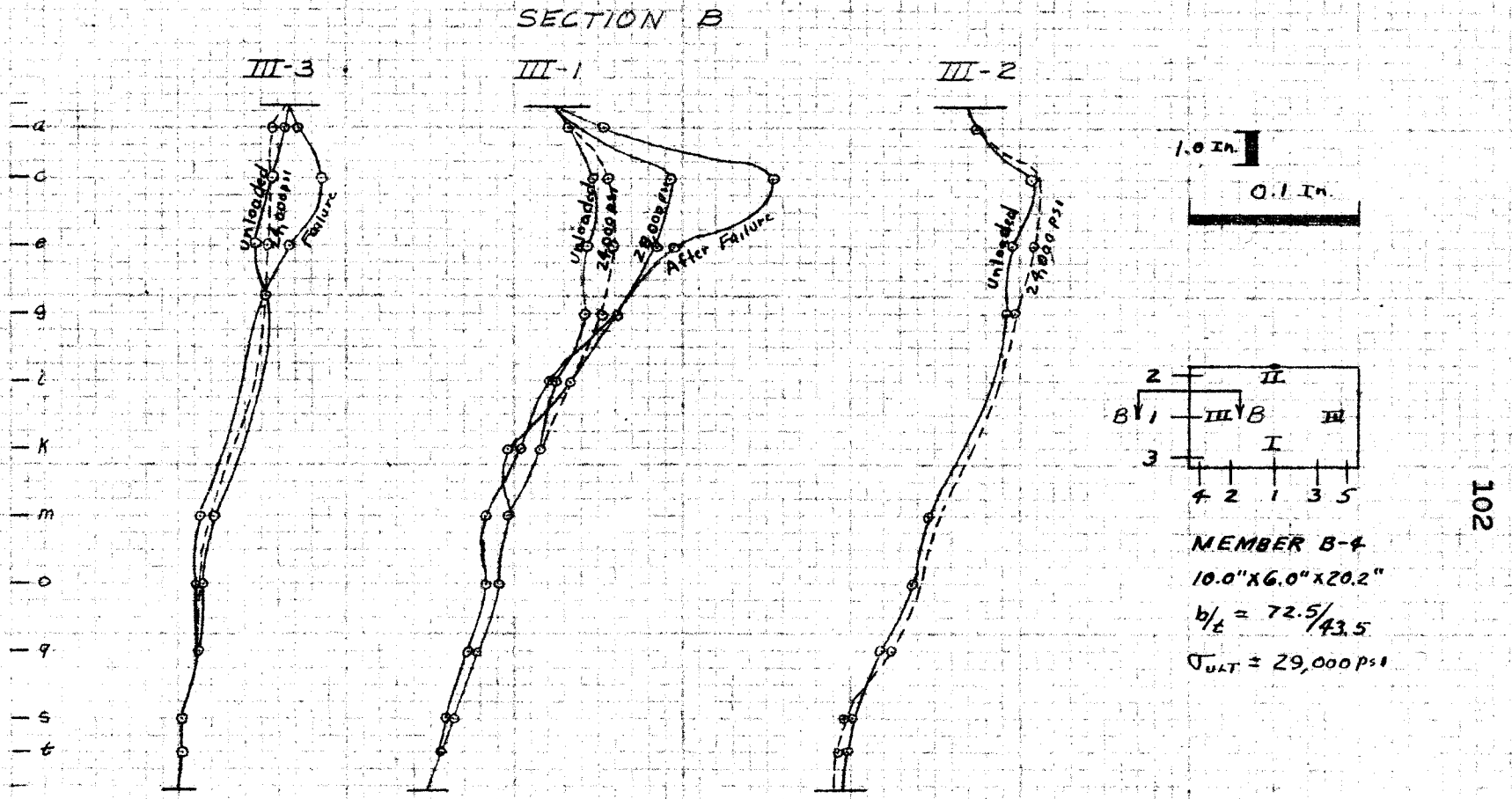


Fig. 22b. Load-Deflection Profiles for Side III, Member B-4.

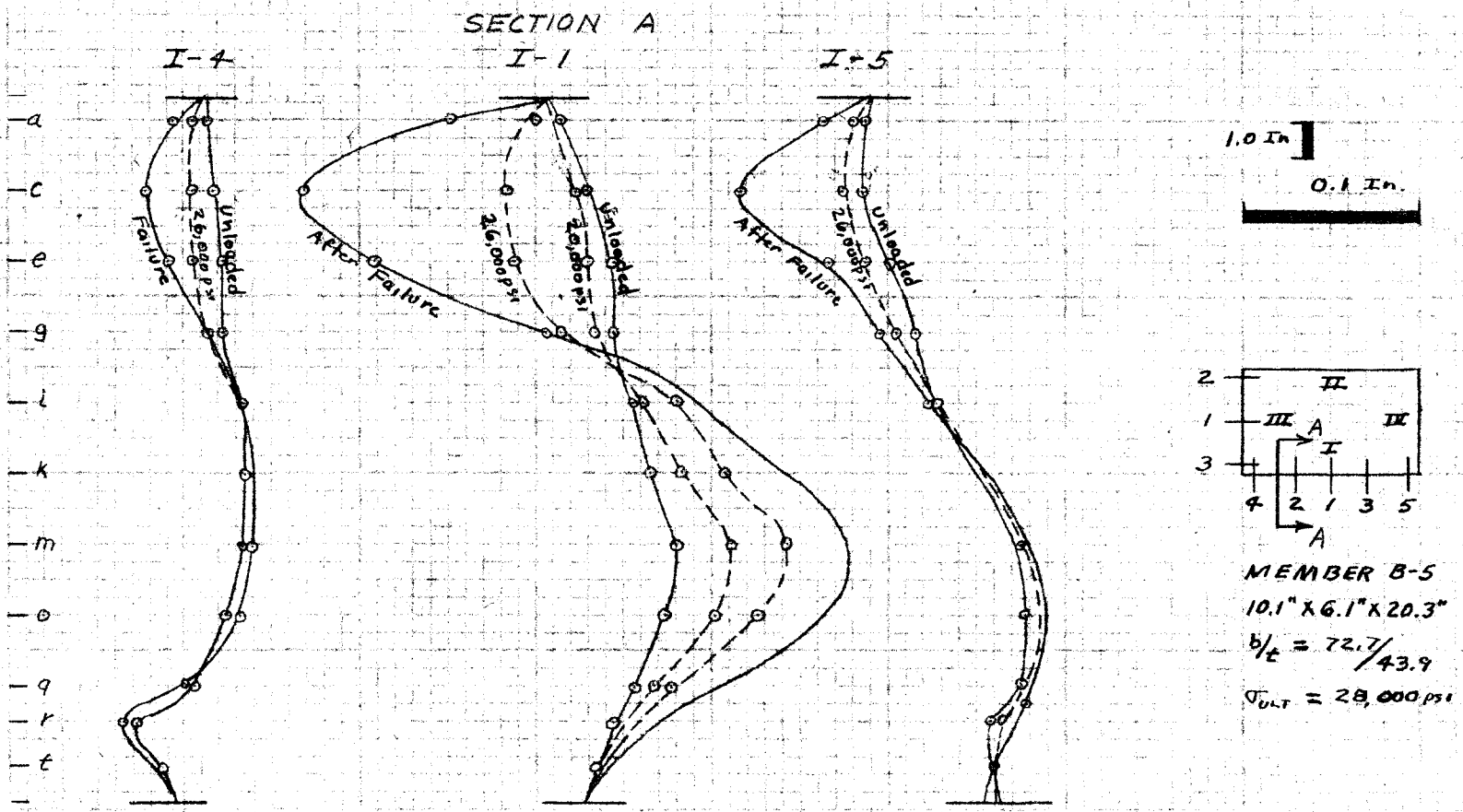


Fig. 23a. Load-Deflection Profiles for Side I, Member B-5.

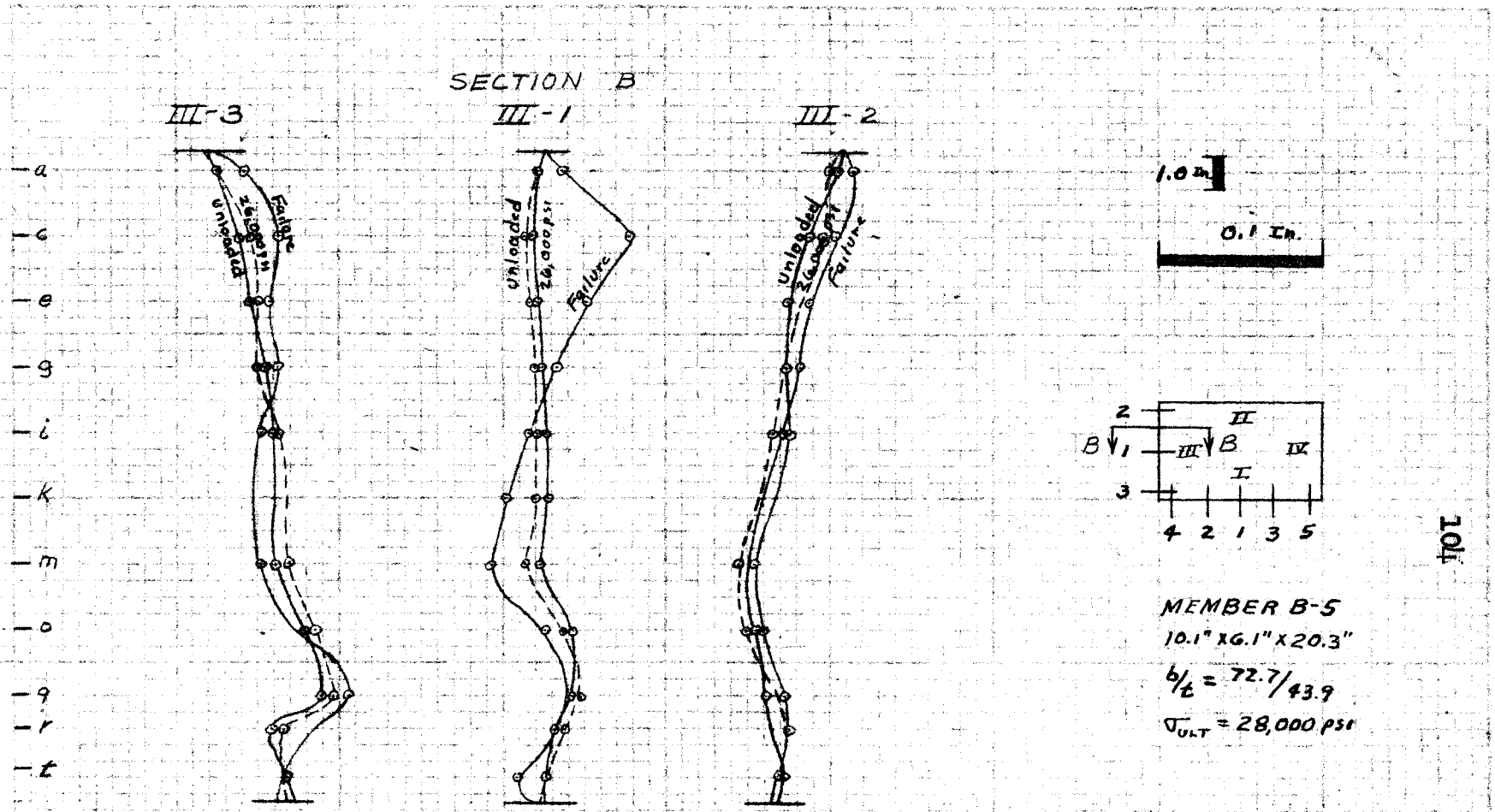


Fig. 23b. Load-Deflection Profiles for Side III, Member B-5.

Form No. 699—10 Squares to Inch
AMERICAN PAD & PAPER CO., HOLYOKE, MASS.

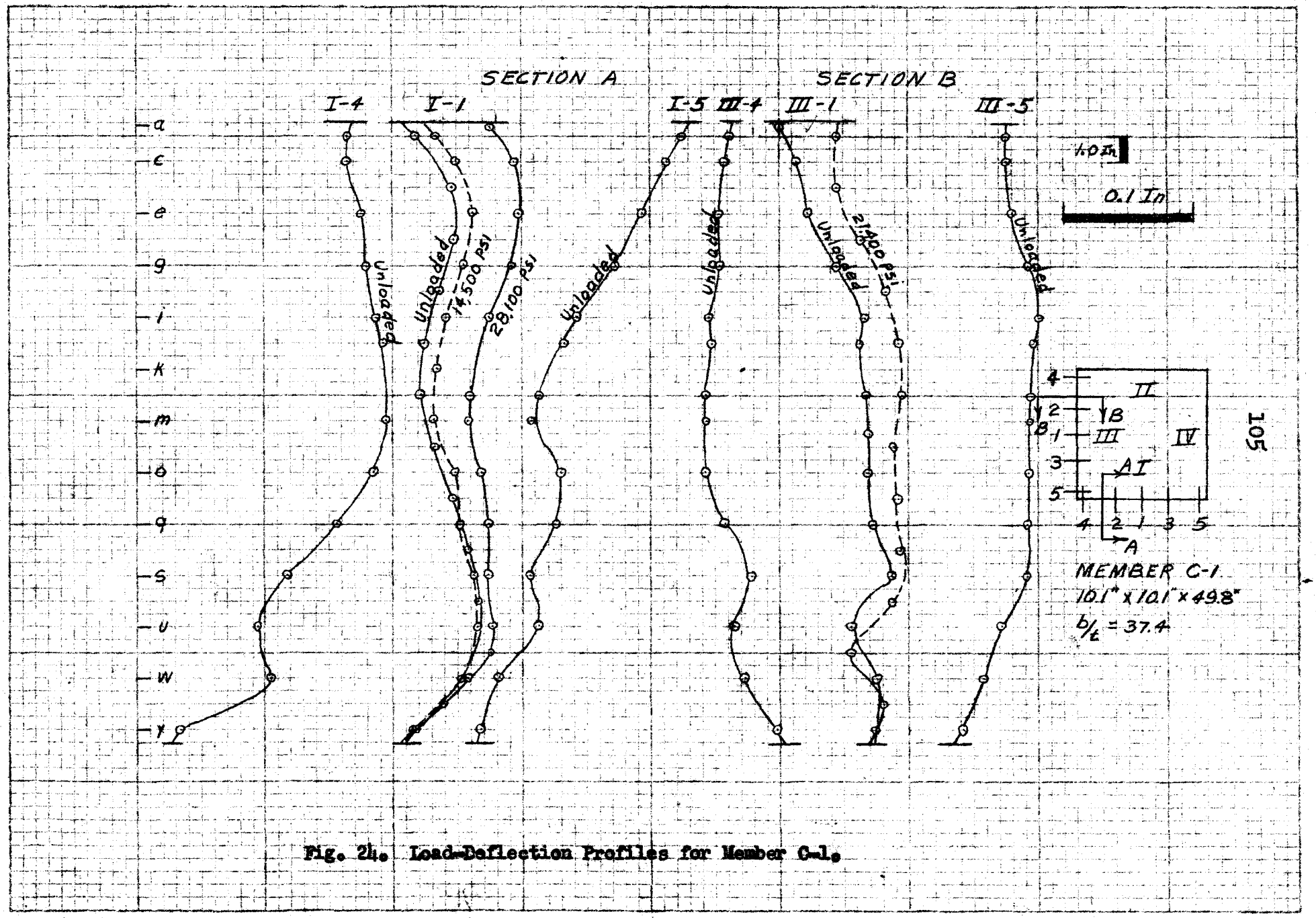


Fig. 24. Load-Deflection Profiles for Member C-1.

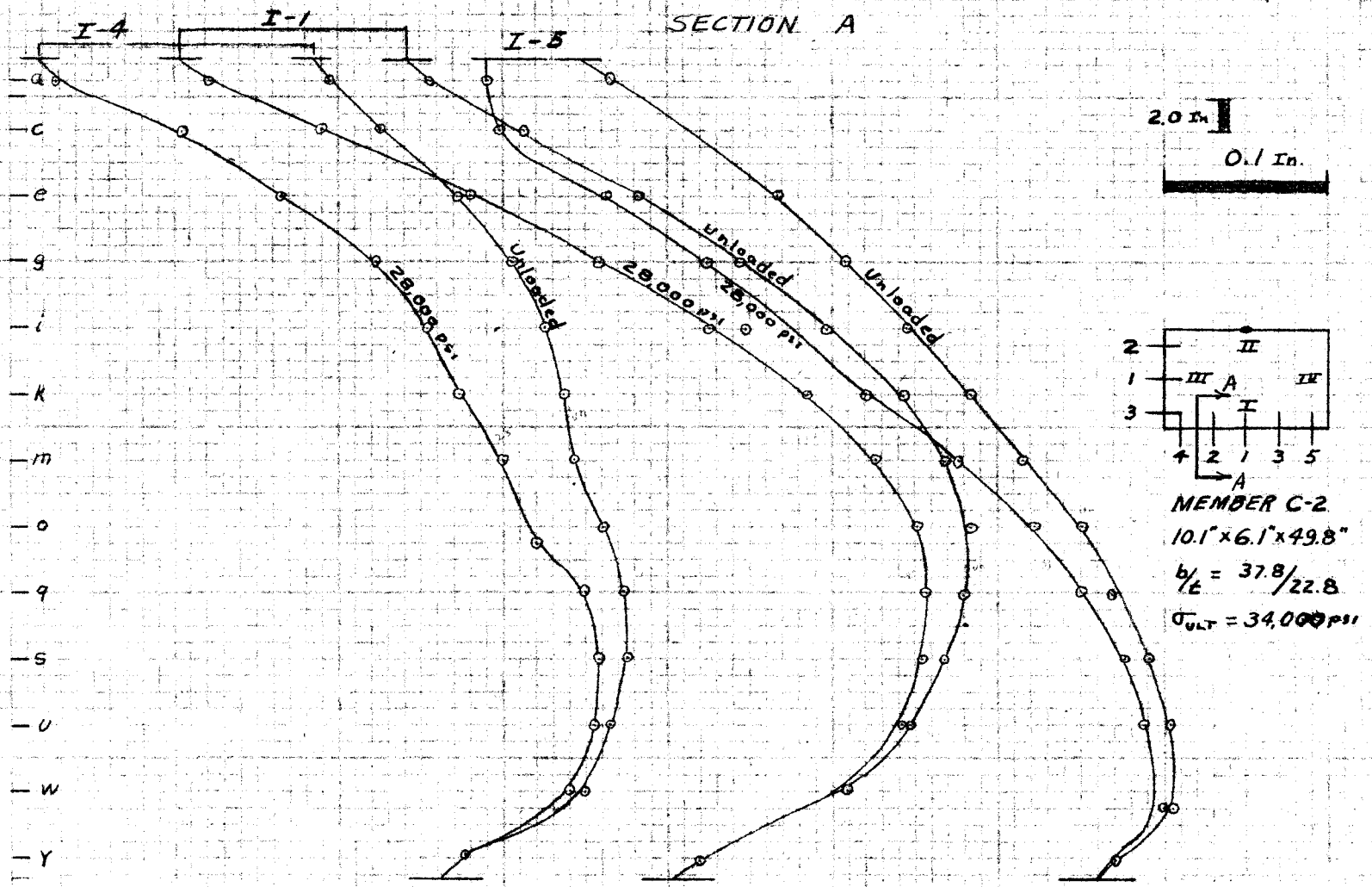


Fig. 25a. Load-Deflection Profiles for Side I, Member C-2.

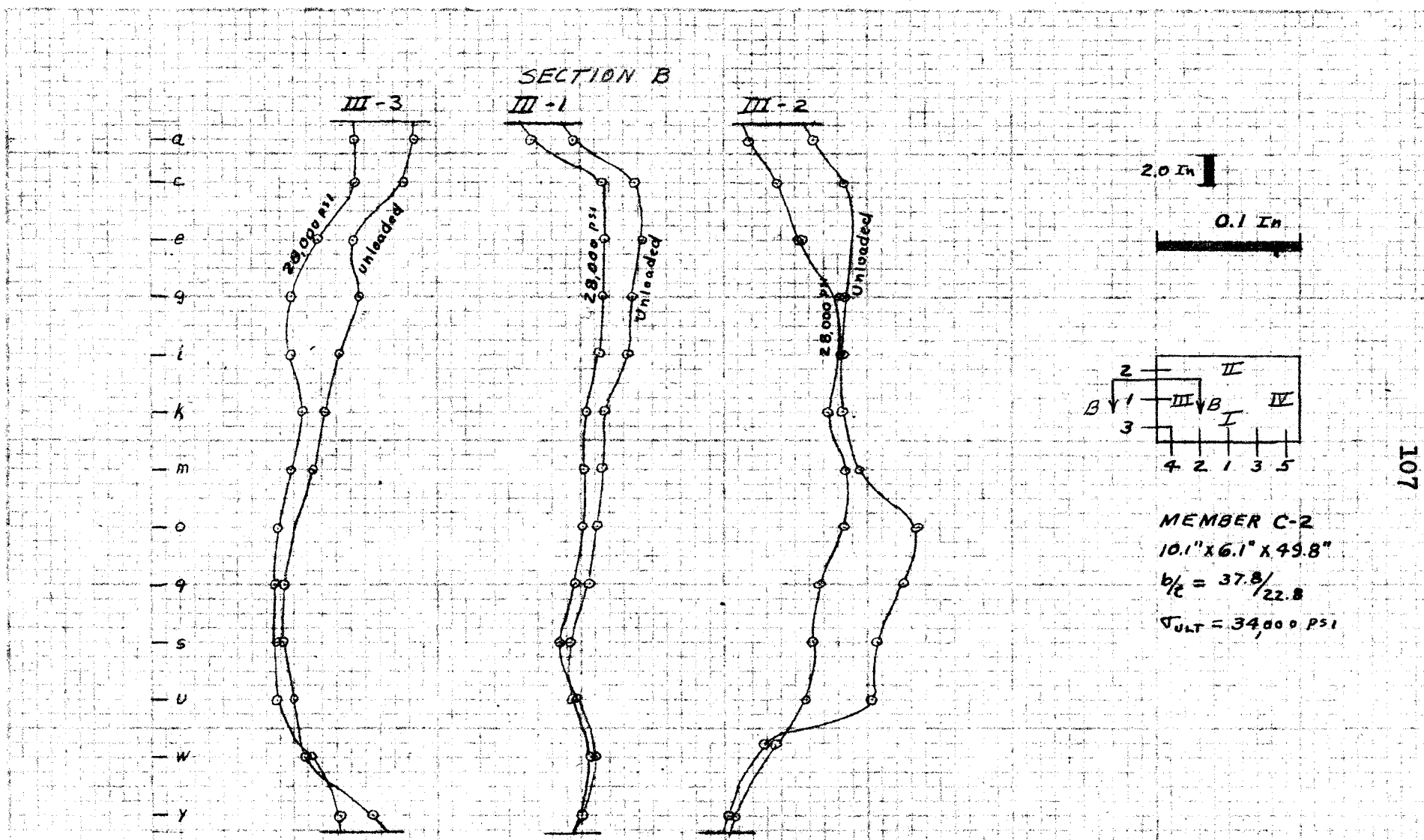


Fig. 25b. Load-Deflection Profiles for Side III, Member C-2.

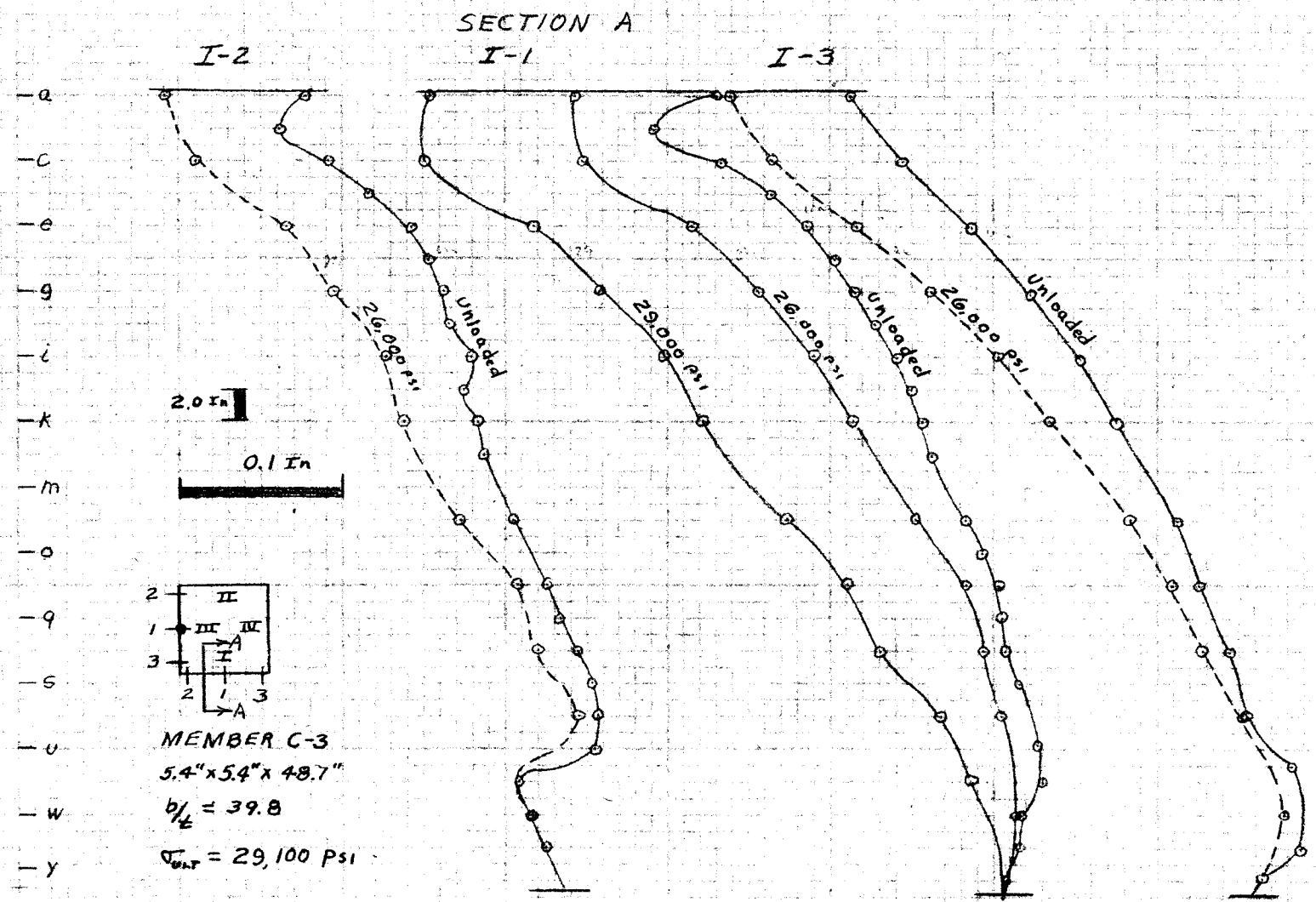


Fig. 26a. Load-Deflection Profiles for Side I, Member C-3.

AMERICAN PAPER TUBE CO., HOLYOKE, MASS.

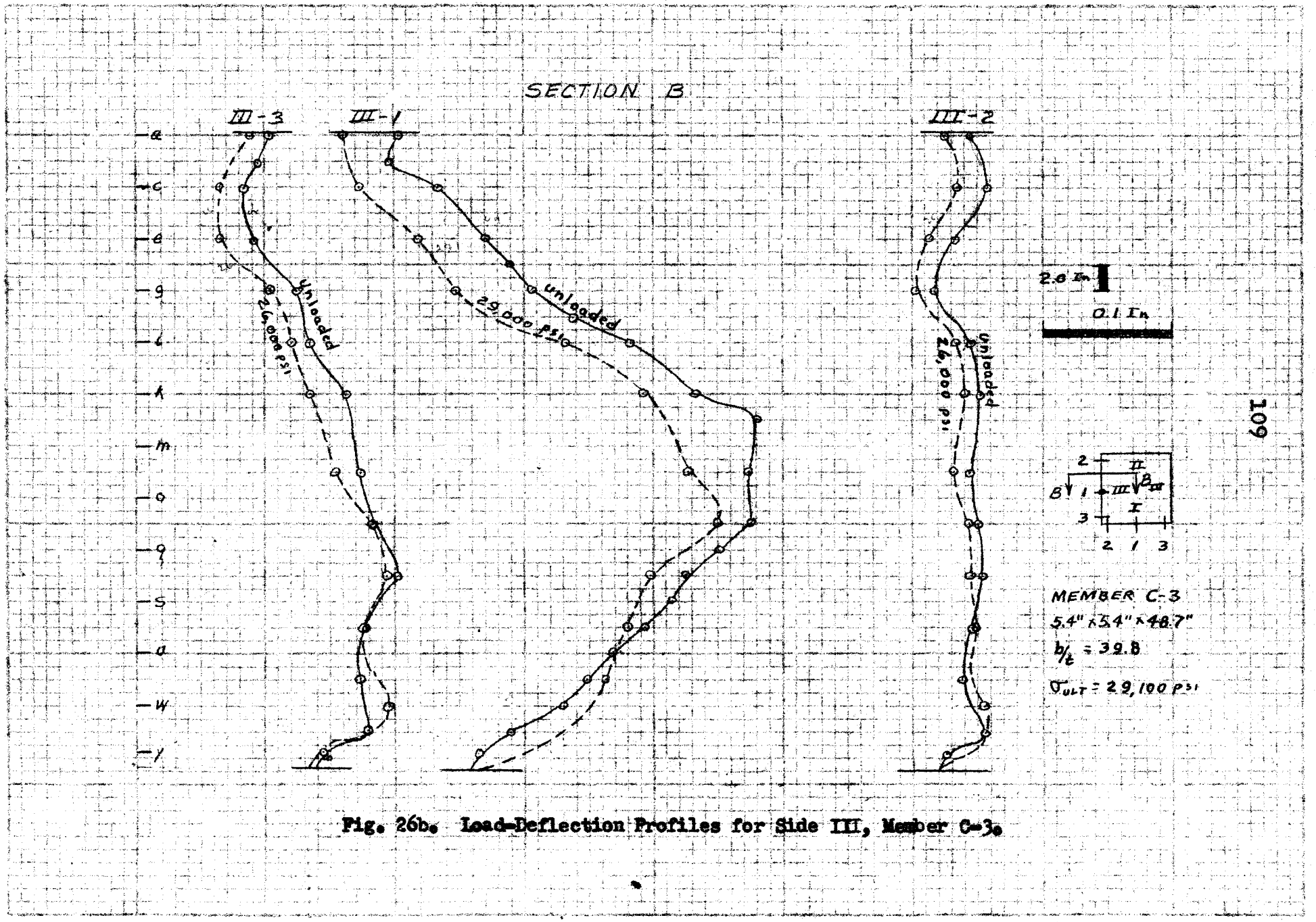


Fig. 26b. Load-Deflection Profiles for Side III, Member C-3.

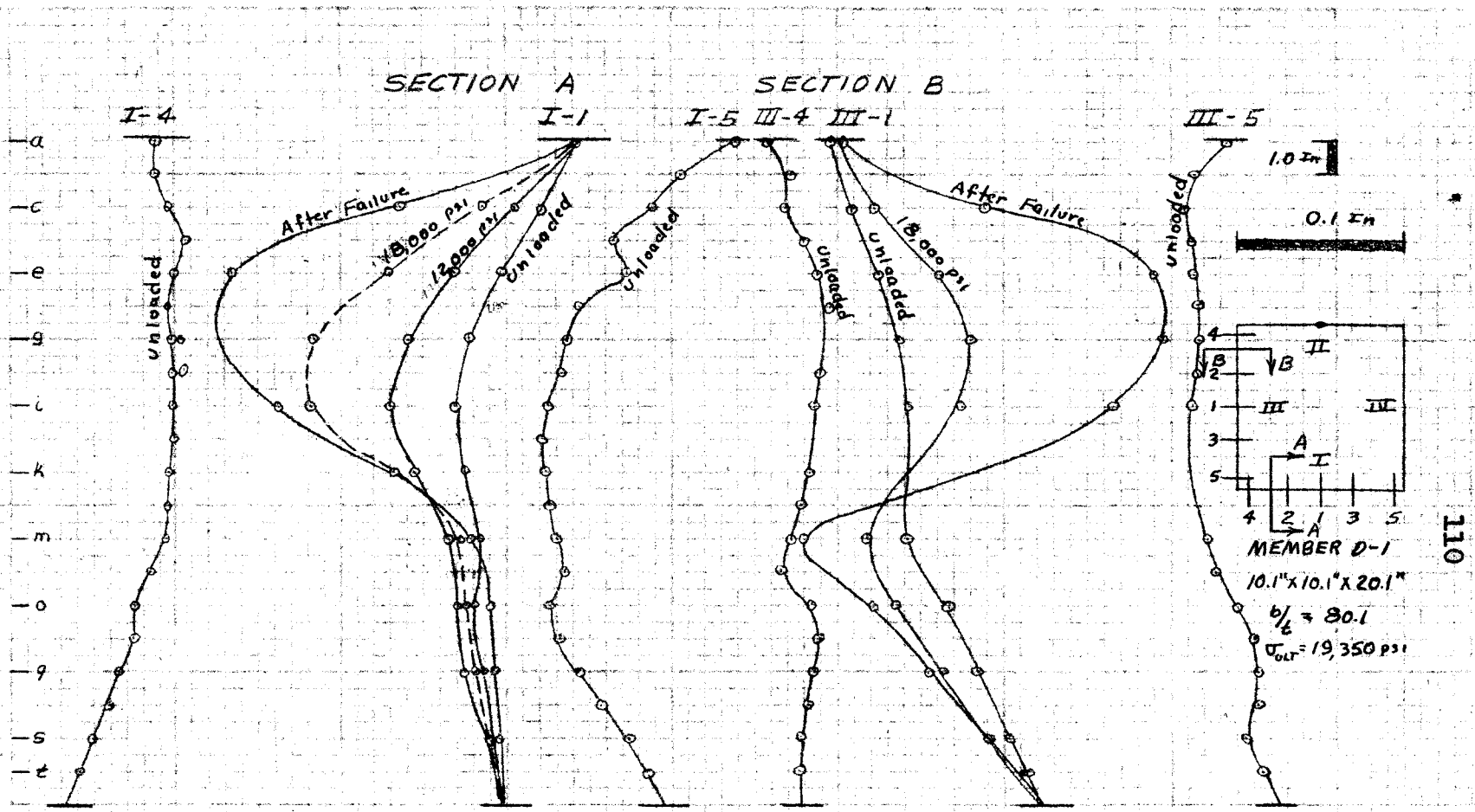


Fig. 27. Load-Deflection Profiles for Member D-1.

AMERICAN PAD & PAPER CO., HOLYOKE, MASS.

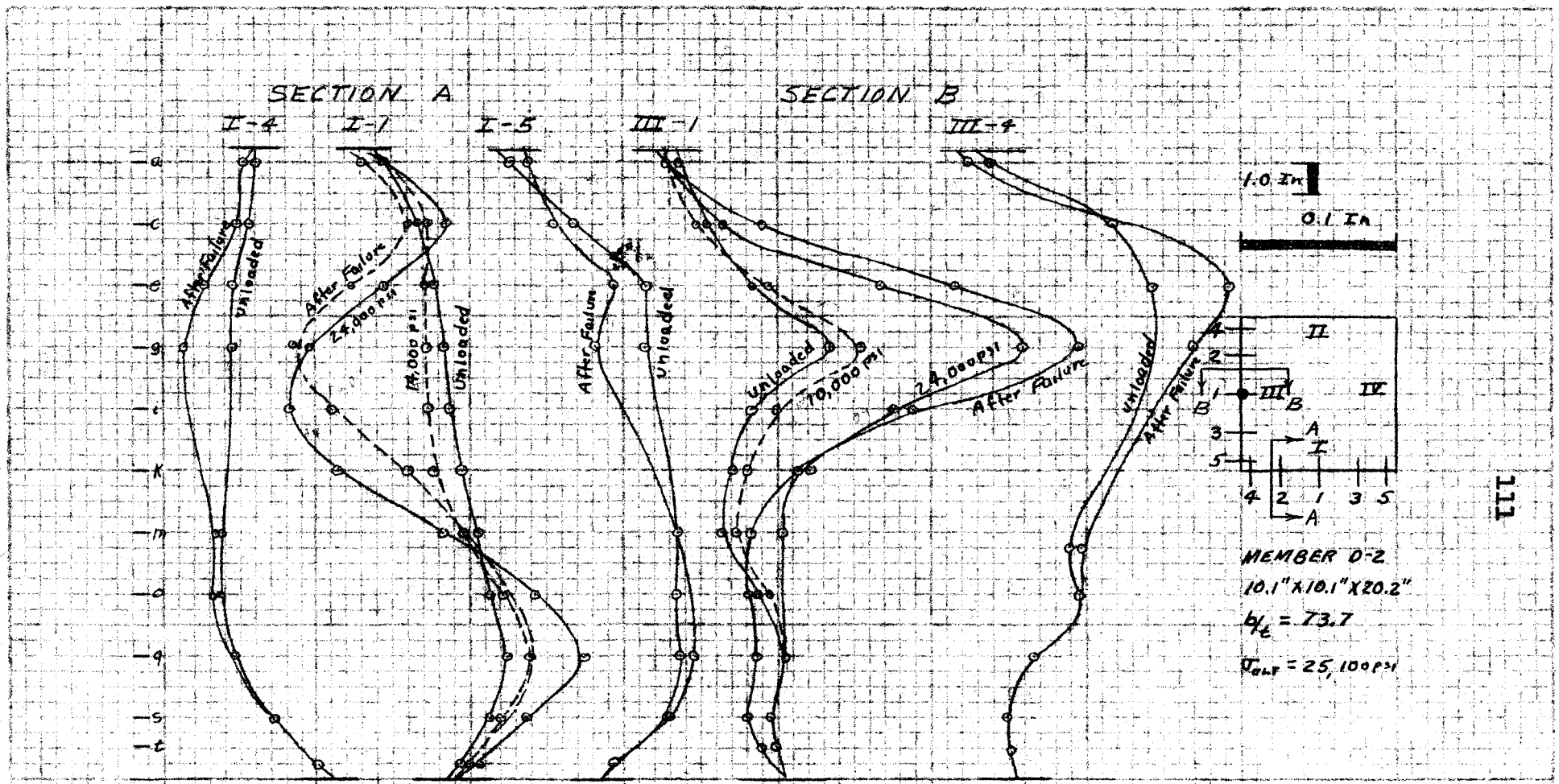


Fig. 28. Load-Deflection Profiles for Member D-2.

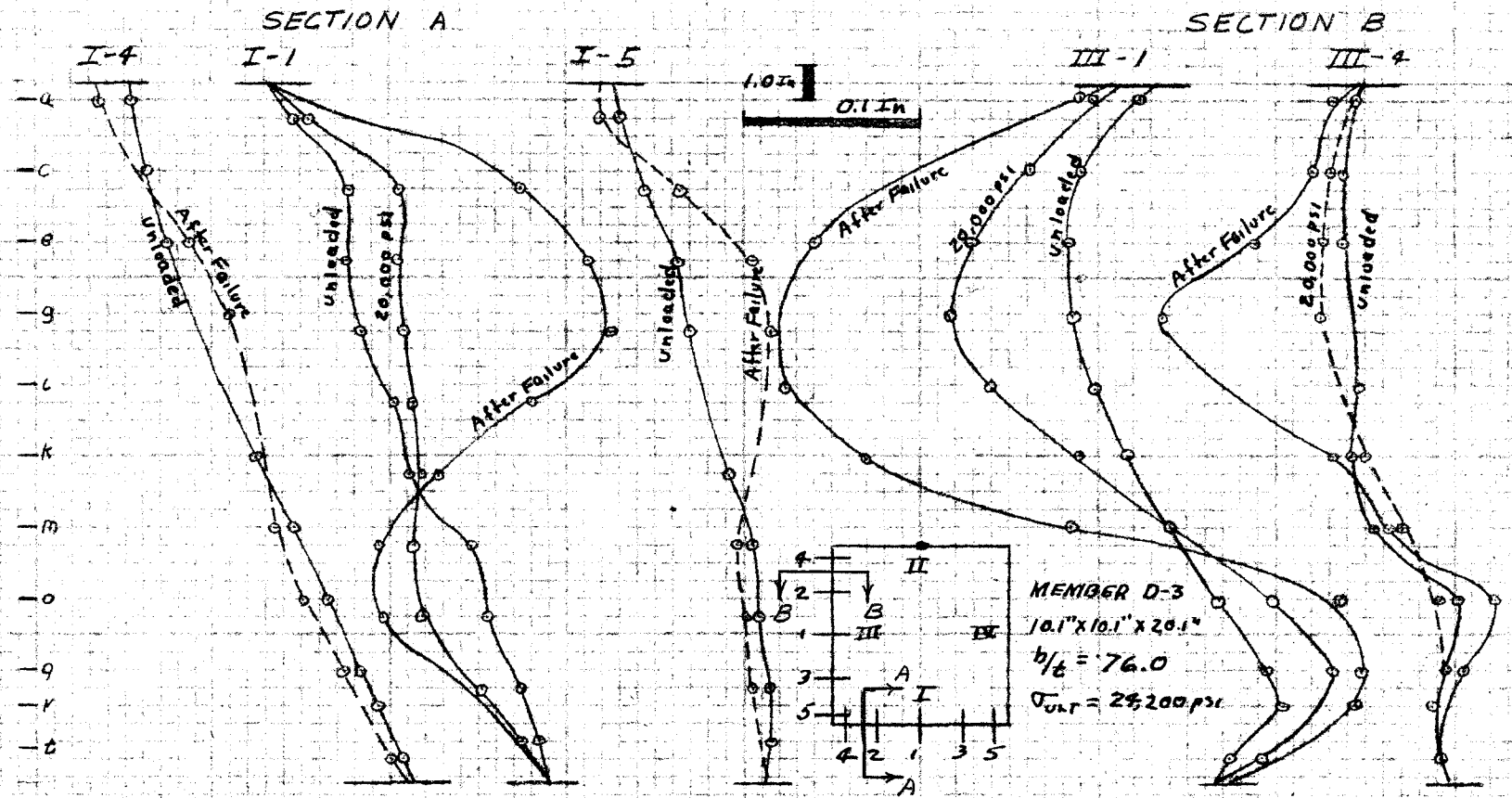


Fig. 29. Load-Deflection Profiles for Member D-3.

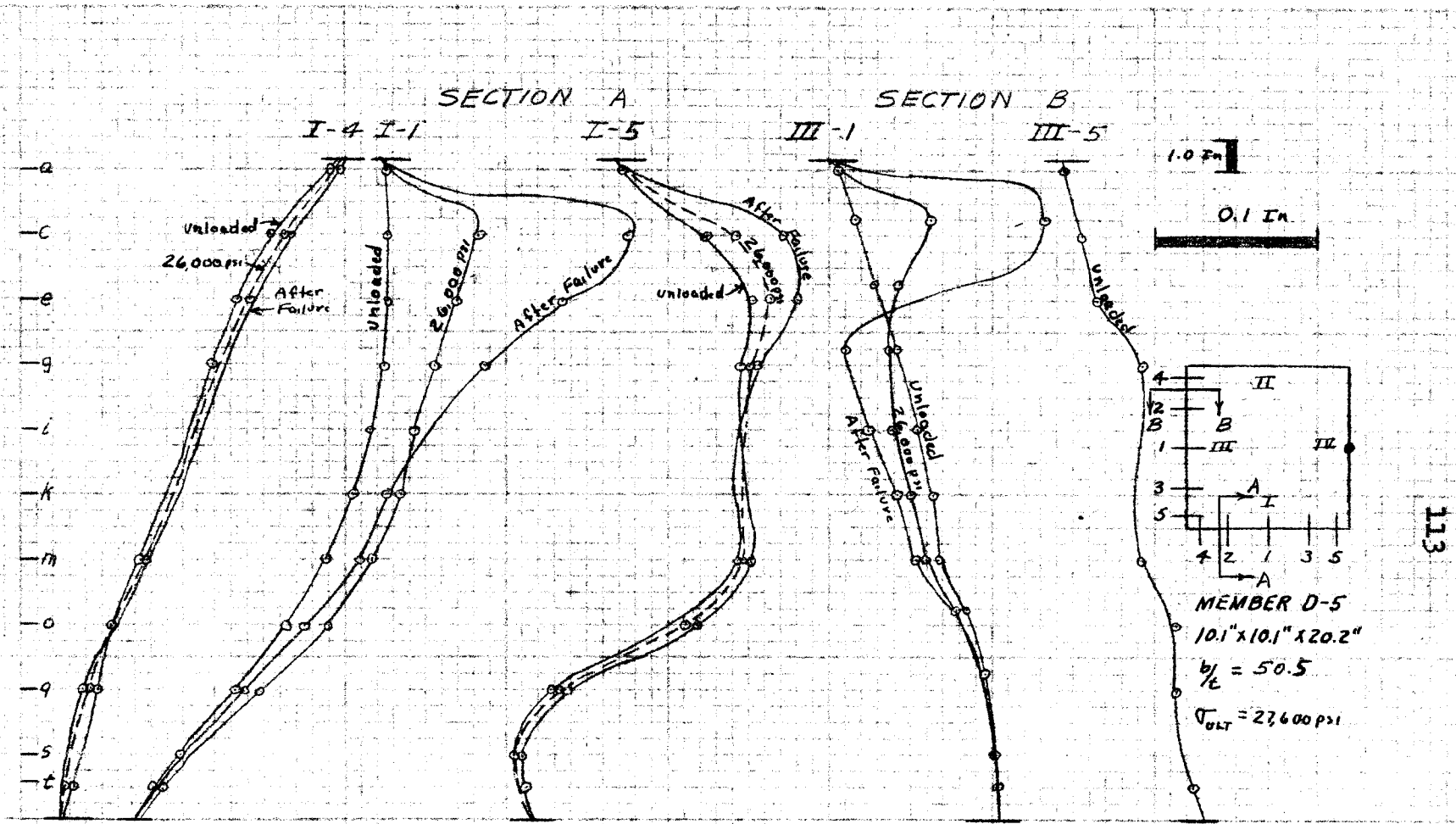


Fig. 30. Load-Deflection Profiles for Member D-5.

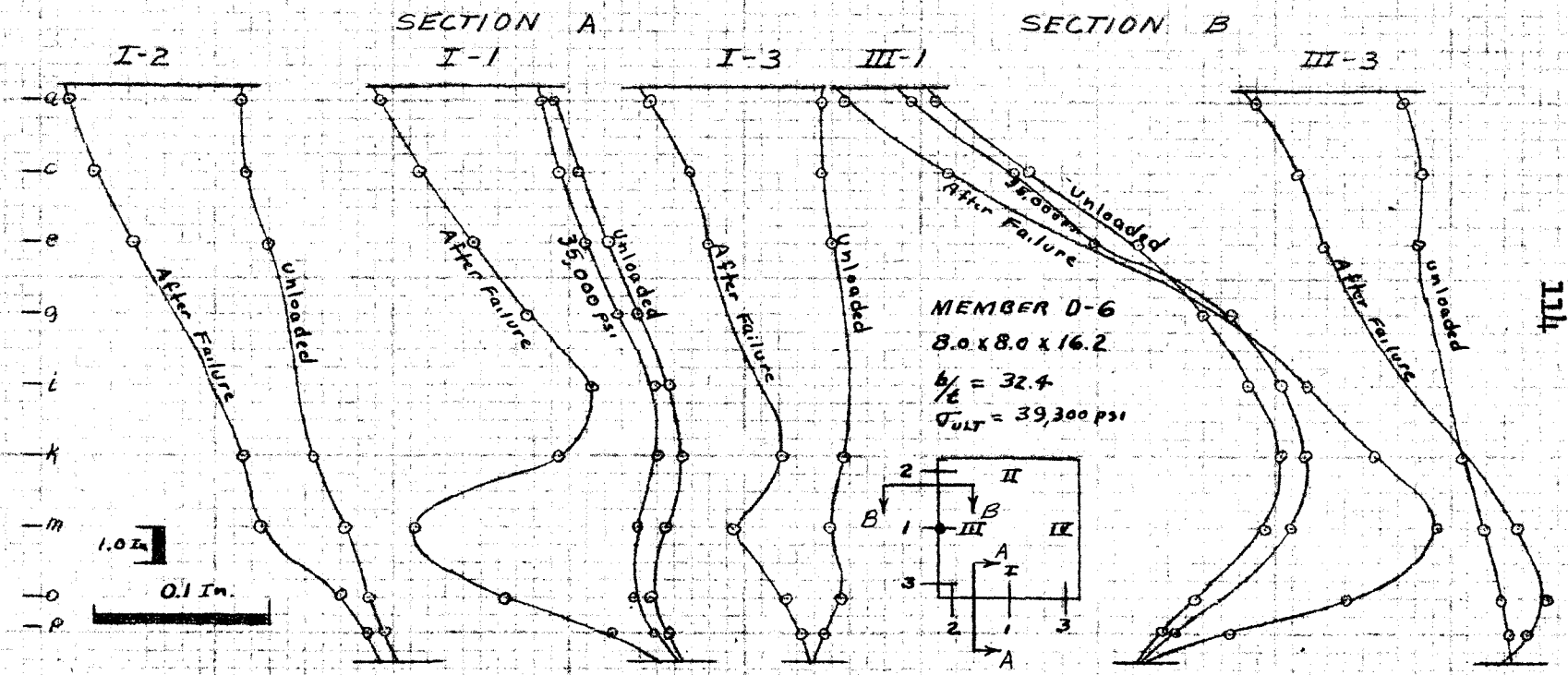


Fig. 31. Load-Deflection Profiles for Member D-6.

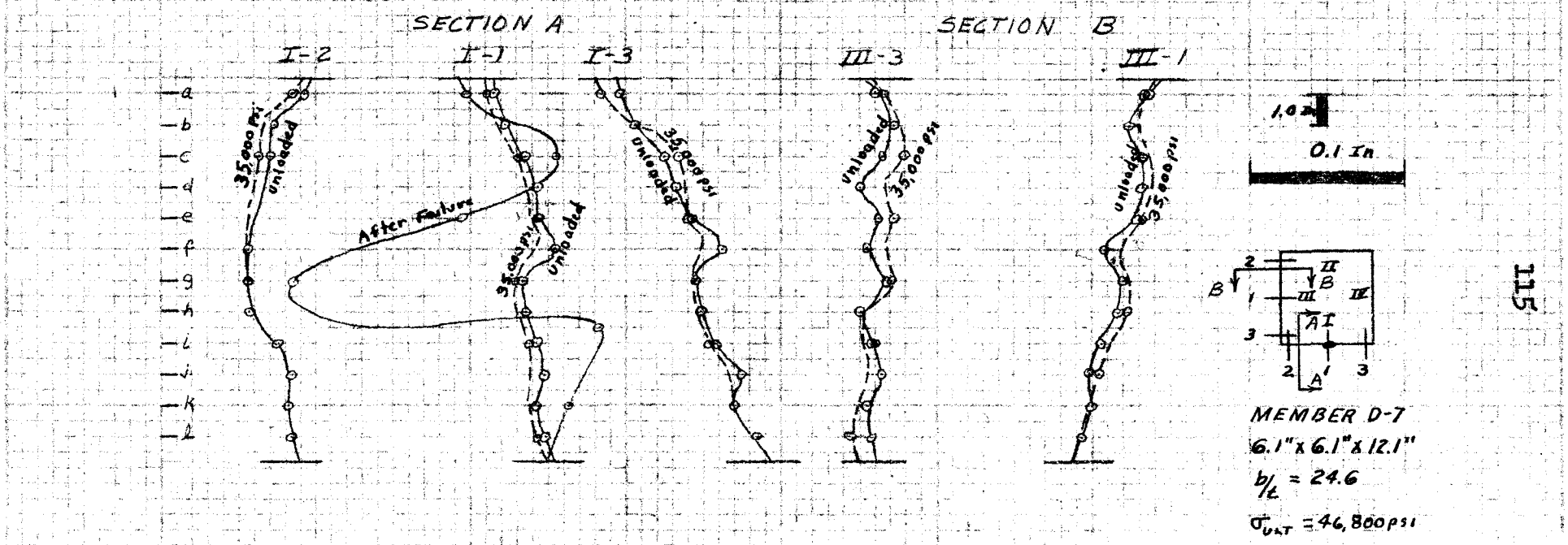


Fig. 32. Load-Deflection Profiles for Member D-7.

B. Load-Deflection Curves for Critical Points on Members Which Failed by Elastic Buckling

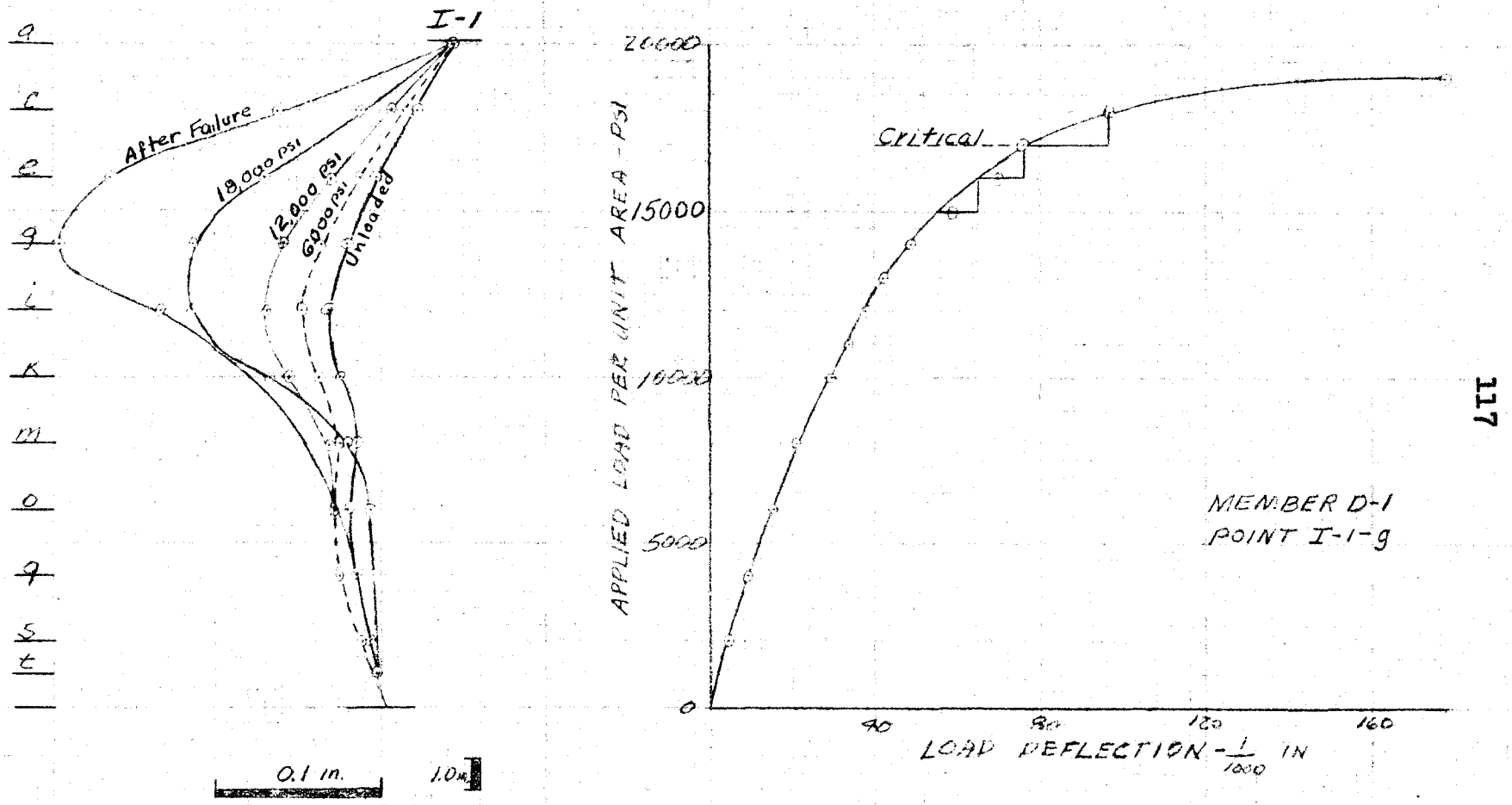


Fig. 33a. Load-Deflection Curve for Critical Point on Side I, Member D-1.

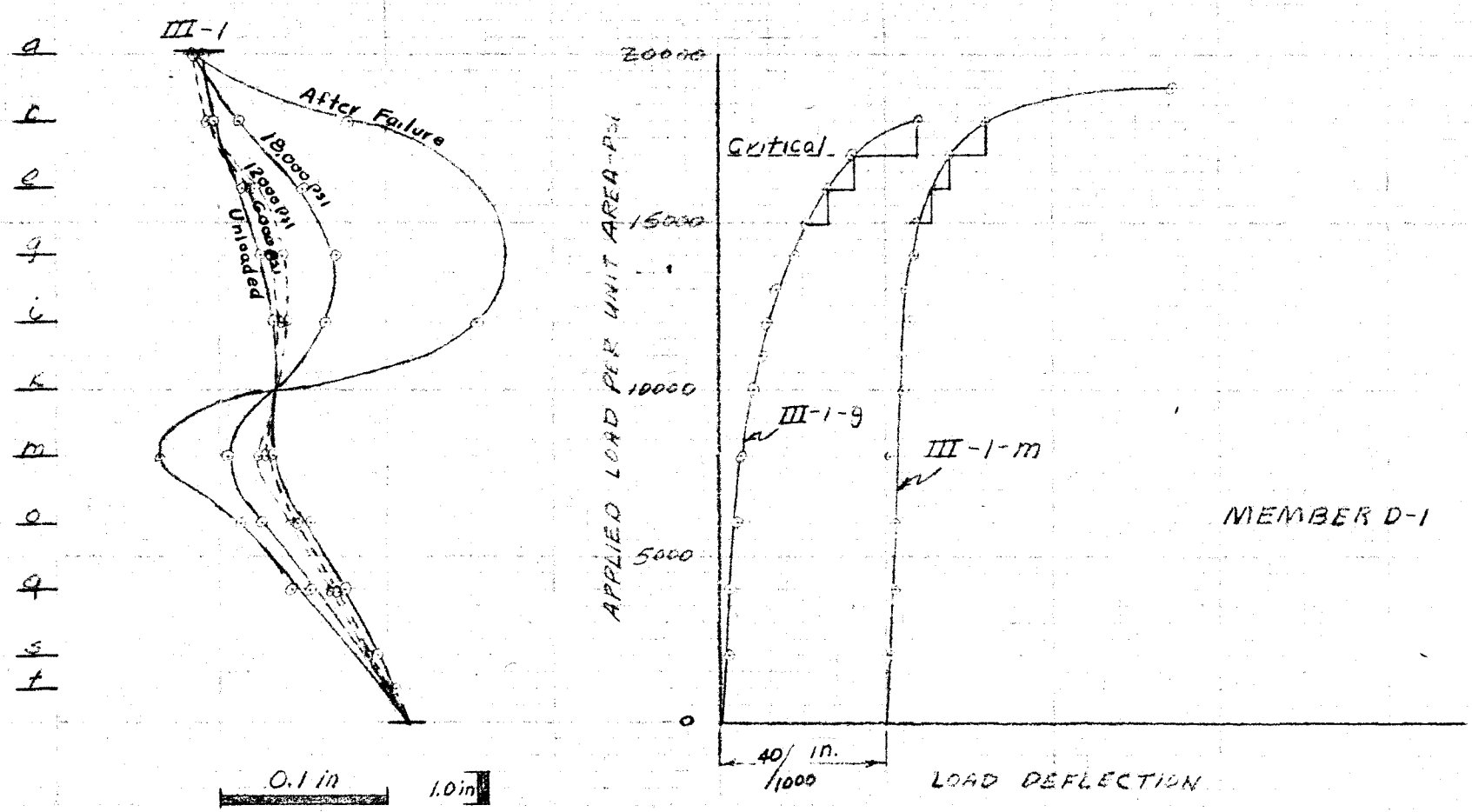


Fig. 33b. Load-Deflection Curves for Critical Points on Side III, Member D-1.

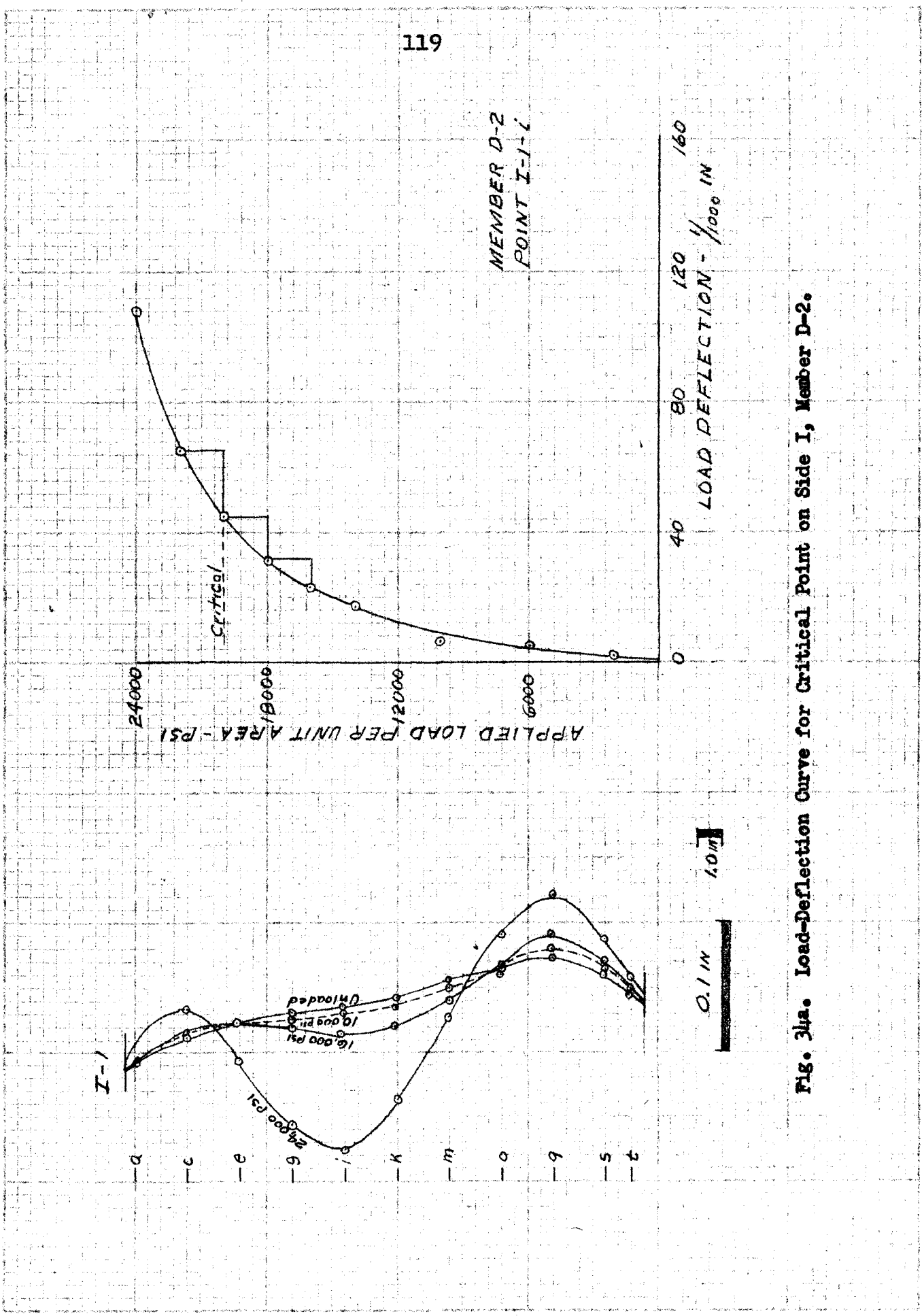
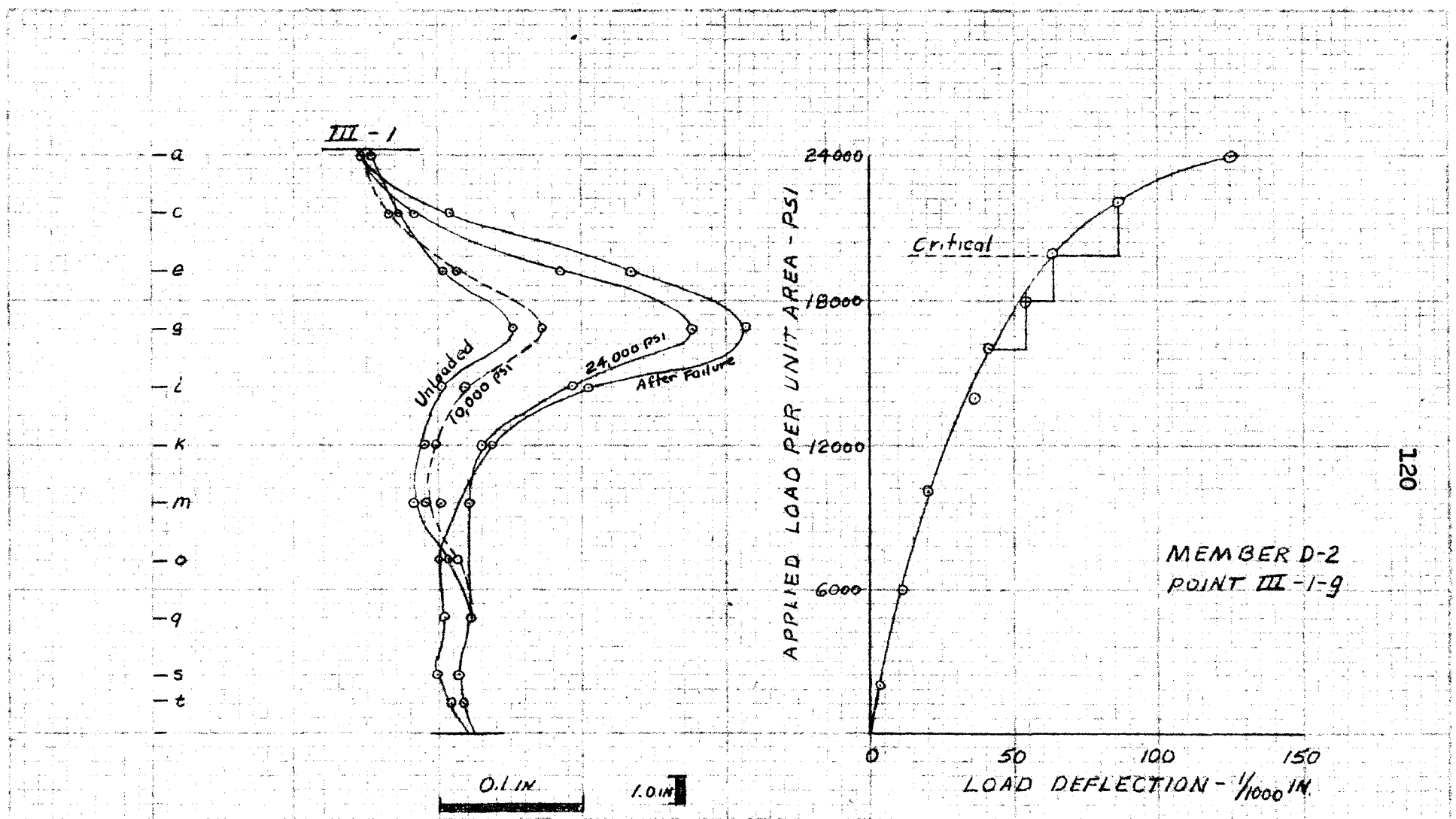


Fig. 34a. Load-Deflection Curve for Critical Point on Side I, Member D-2.



120

Fig. 34b. Load-Deflection Curve for Critical Point on Side III, Member D-2.

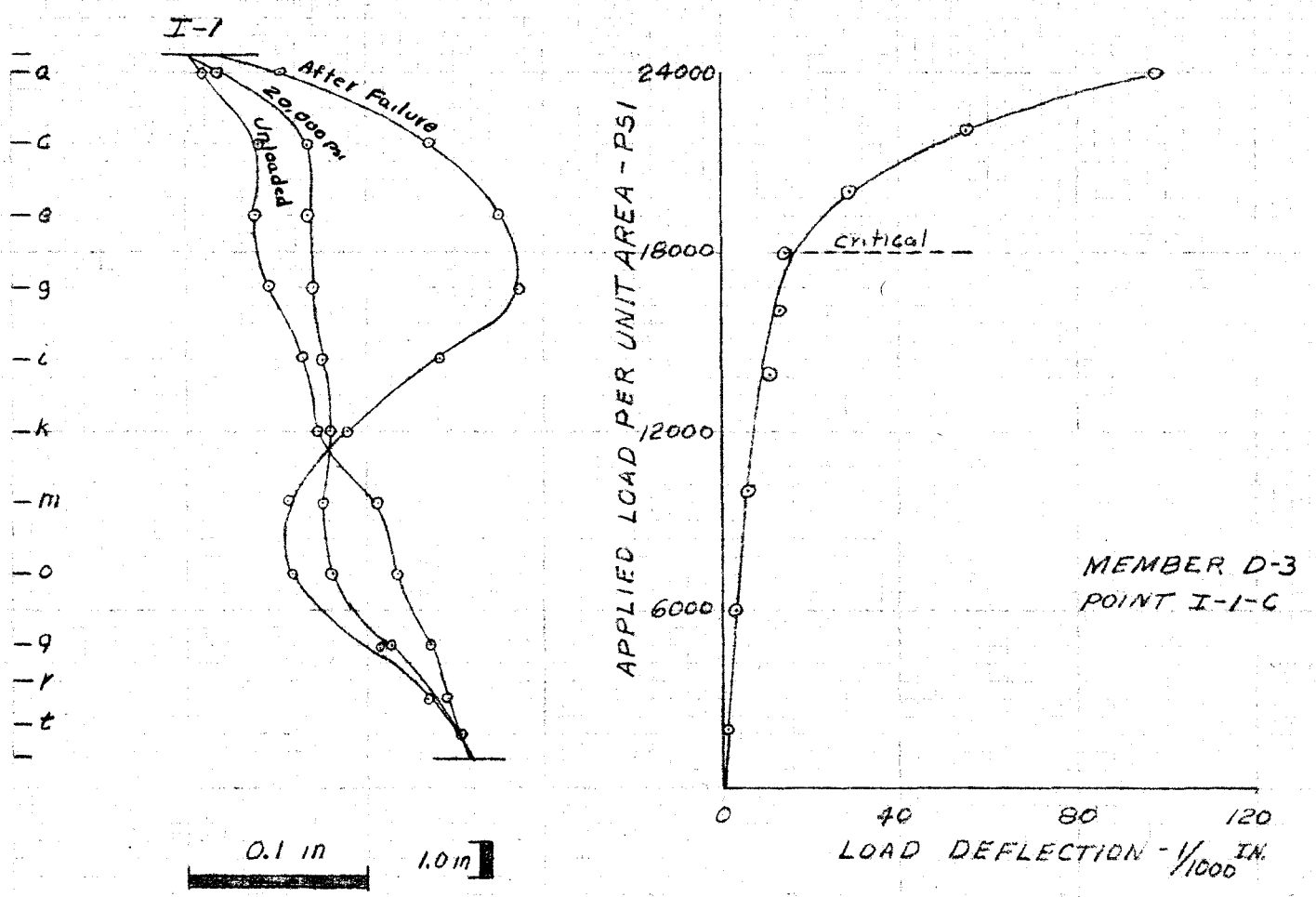


Fig. 35a. Load-Deflection Curve for Critical Point on Side I, Member D-3.

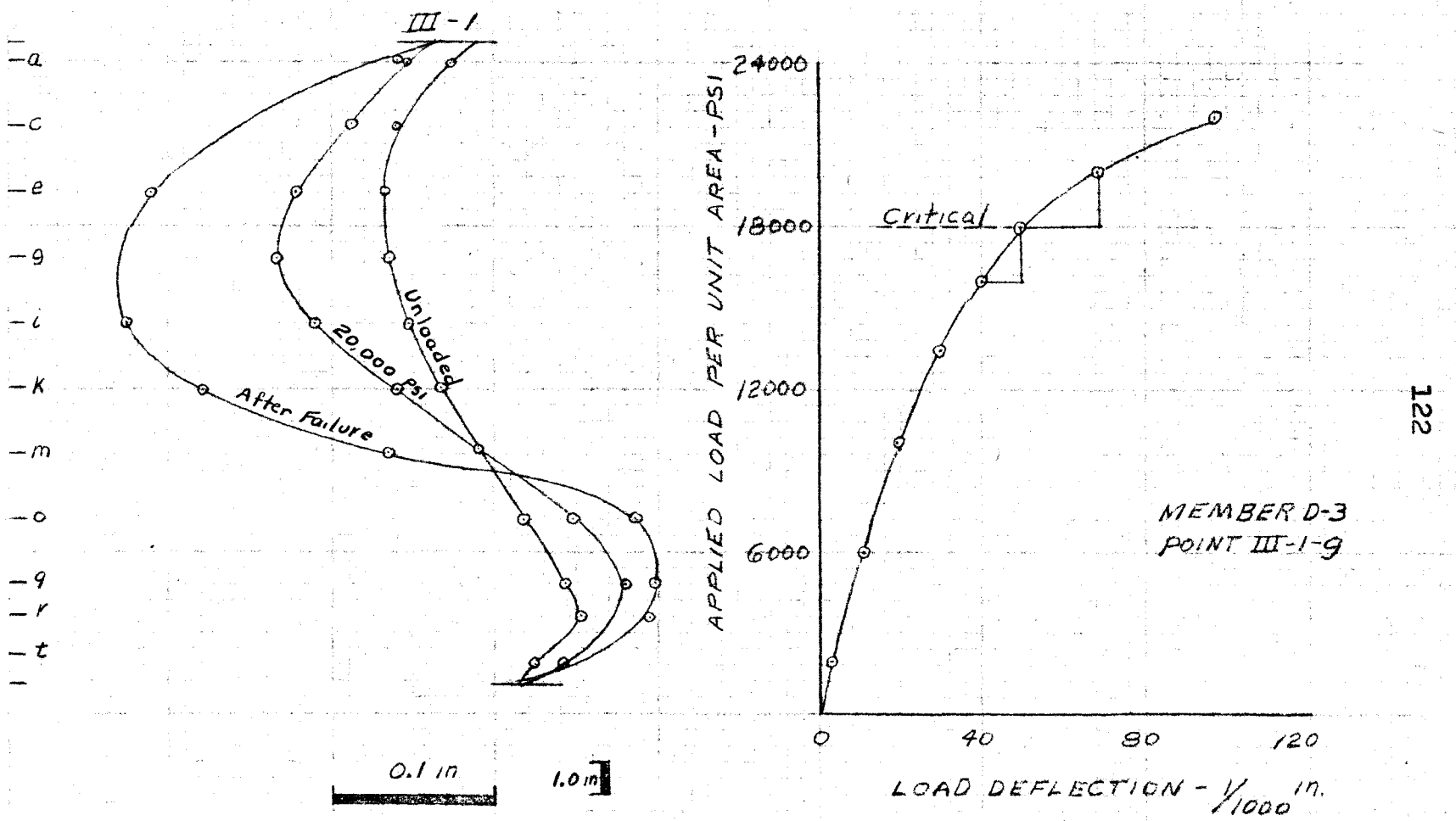


Fig. 35b. Load-Deflection Curve for Critical Point on Side III, Member D-3.

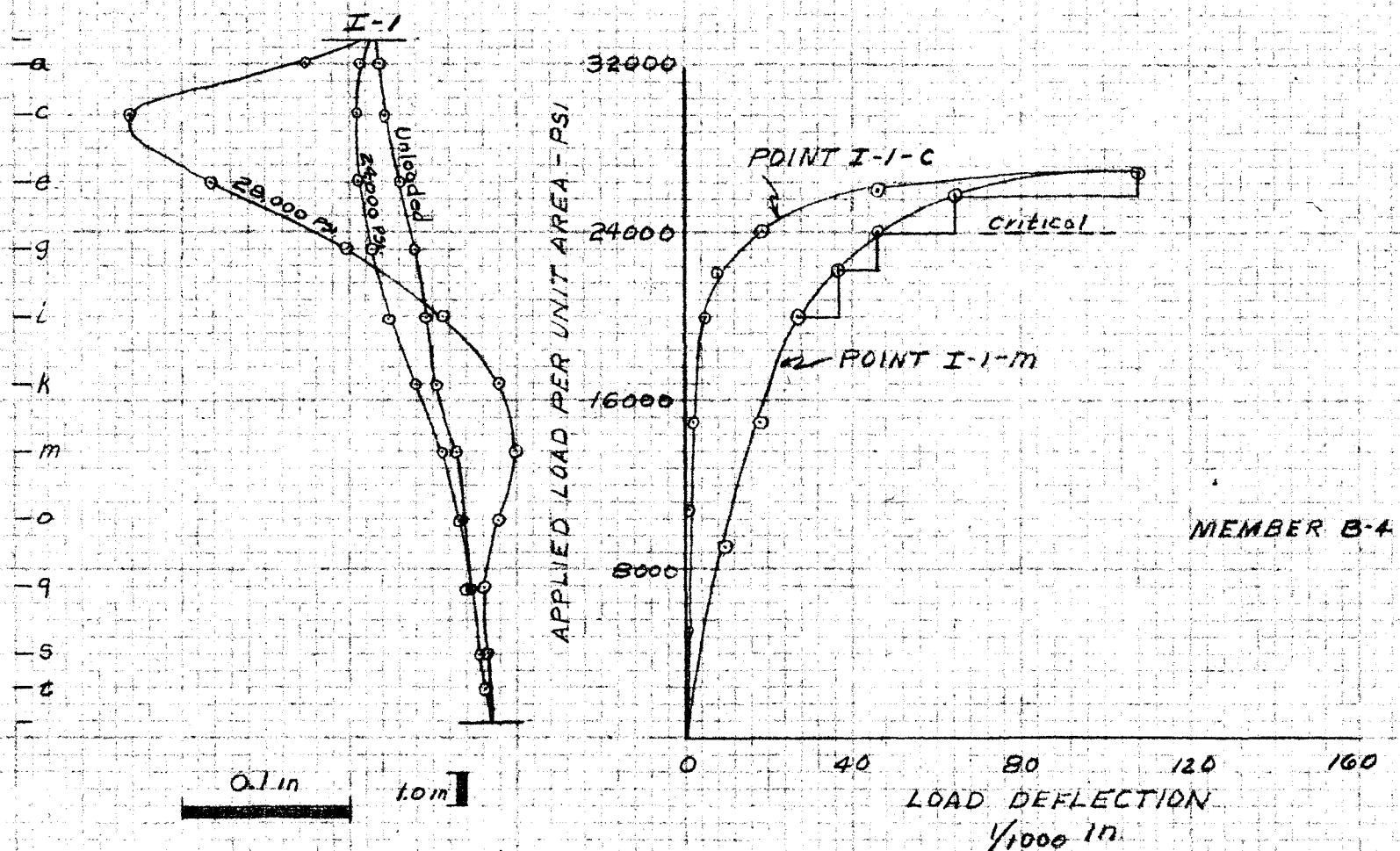


Fig. 36. Load-Deflection Curves for Critical Points on Side I, Member B-4.

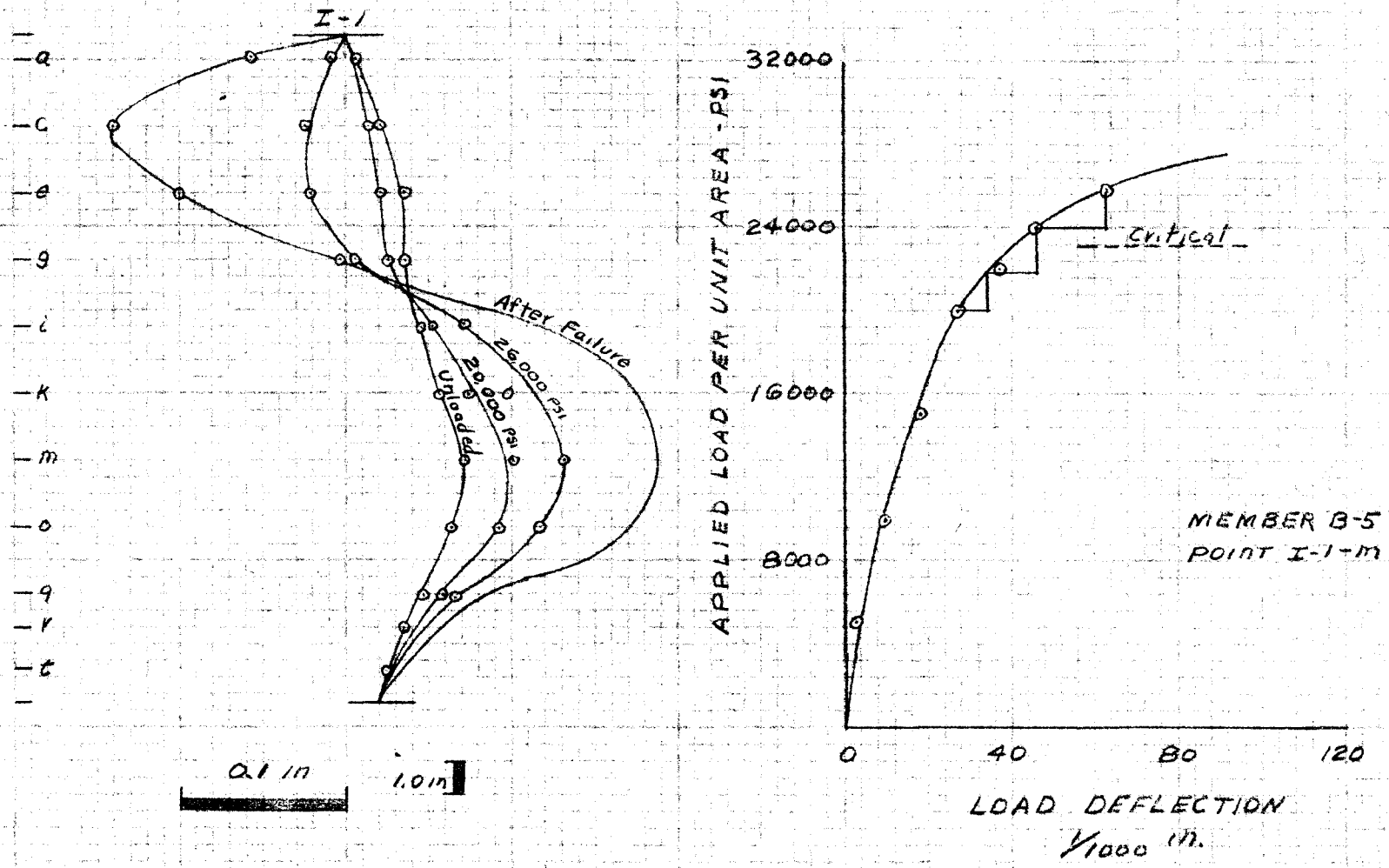
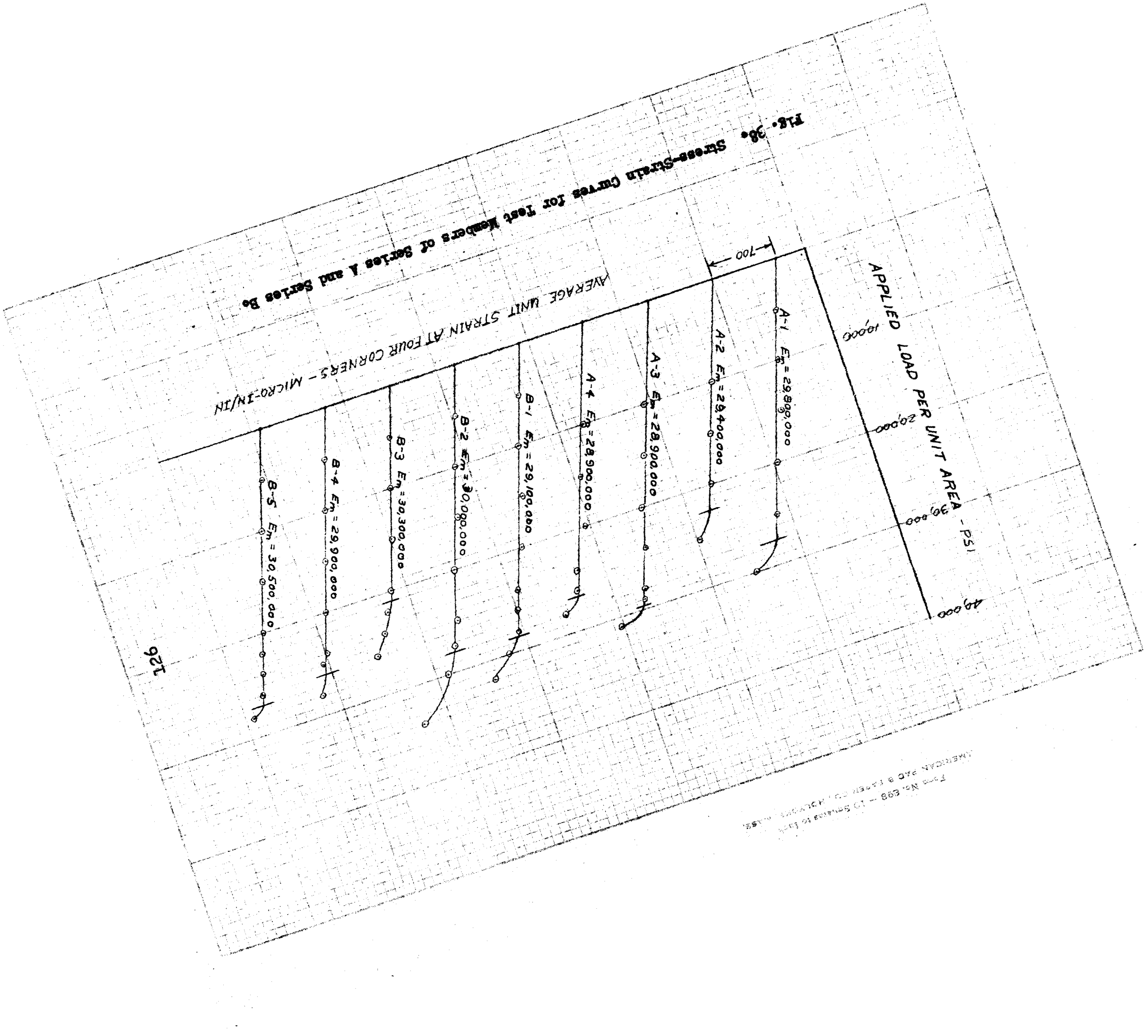


Fig. 37. Load-Deflection Curve for Critical Point on Side I, Member B-5.

C. Stress-Strain Curves for All Members

Fig. 38. Stress-Strain Curves for Test Members of Series A and Series B.



Form No. 8C2 -- 10 Squares to Inch
AMERICAN PAD & PAPER CO., HOLYOKE, MASS.

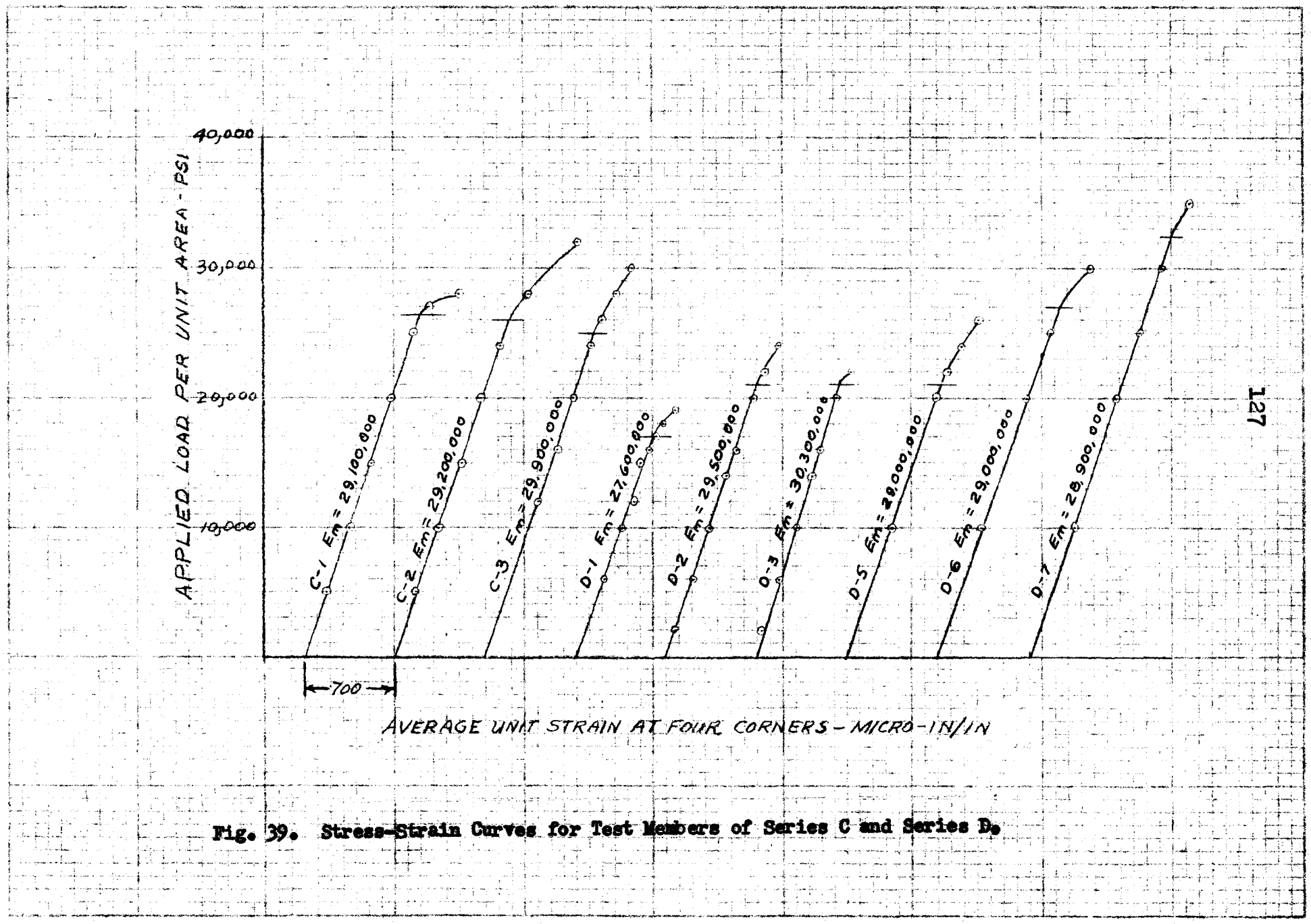


Fig. 39. Stress-Strain Curves for Test Members of Series C and Series D.

D. Plate Contour Meter

Plate Contour Meter

The plate contour meter consists essentially of a base, a fixed vertical assembly, a main sliding assembly, and a deflection-measuring mechanism. The main sliding assembly is guided on the vertical assembly by two dovetailed runners and is fixed in position at any point along these runners by three small cap screws with brass plug tips beveled to fit the sloping surface of the dovetail. This sliding assembly has two arms mutually perpendicular in a horizontal plane, each of which guides a small slide on which a depth-gage micrometer is mounted. These slides are 3½ inches long and have a gib and set screw arrangement which allows a micrometer reading to be taken anywhere along a 13-inch length of the arm. Deflection measurements can be made to the nearest 1/1,000 inch.

The base rests on three points, a pivotal plug, and a 5/8-inch set screw at the end of each leg of the base. These set screws are used to plumb the vertical assembly. Small steel plates are used to protect the bed of the testing machine. The 70-inch high vertical assembly is fastened to the base by four 5/8-inch cap screws.

Counterbalancing of the main sliding assembly is accomplished by using a 1/8-inch airplane cable, a 52-lb. lead cylinder, and a ball-bearing sheave which is mounted on top of the vertical assembly.

Each micrometer is equipped with six rods which give a range of from 0 to 6 inches for deflection measurement. This feature in addition to the 13-inch range of the micrometer slides and the 65-inch range of the main sliding assembly gives considerable flexibility to the sizes of members which can be accommodated. Each micrometer rod is beveled at the end to give a tip diameter of 1/32-inch. Small pointers mounted on the micrometer slides and the main sliding assembly allow a convenient system of coordinates to be set up for any member.

The base, vertical assembly, and main sliding assembly were constructed of mild structural steel. Preparatory to welding, this steel was cut to shape in the machine shops of the Department of Mechanical Engineering at Iowa State College. The electric-arc welding, annealing, and final machining was done by the National Machine Company, Des Moines, Iowa. The micrometer slides were made in the Engineering Experiment Station Machine Shop.

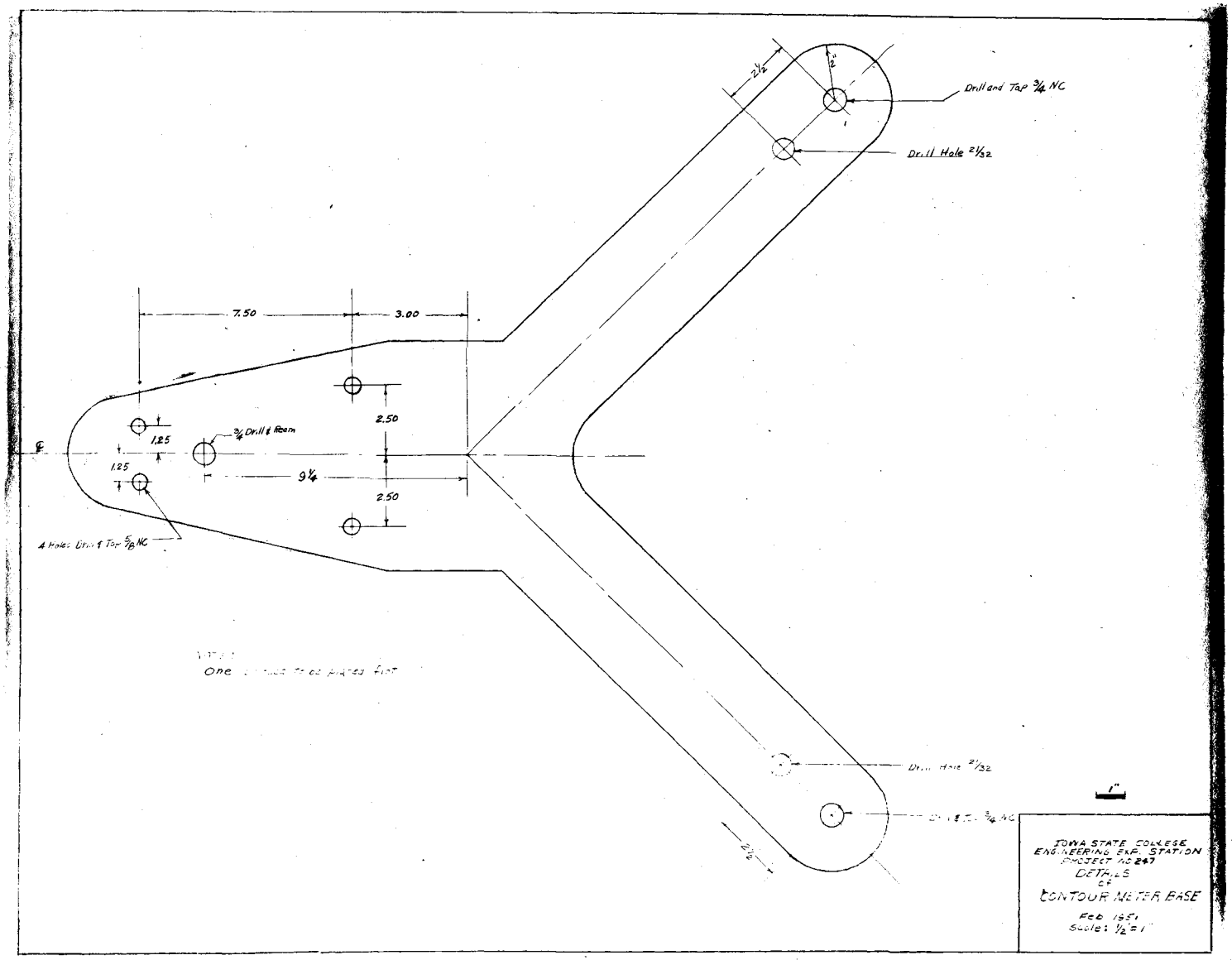


Fig. 40. Details of Base of Plate Contour Meter.

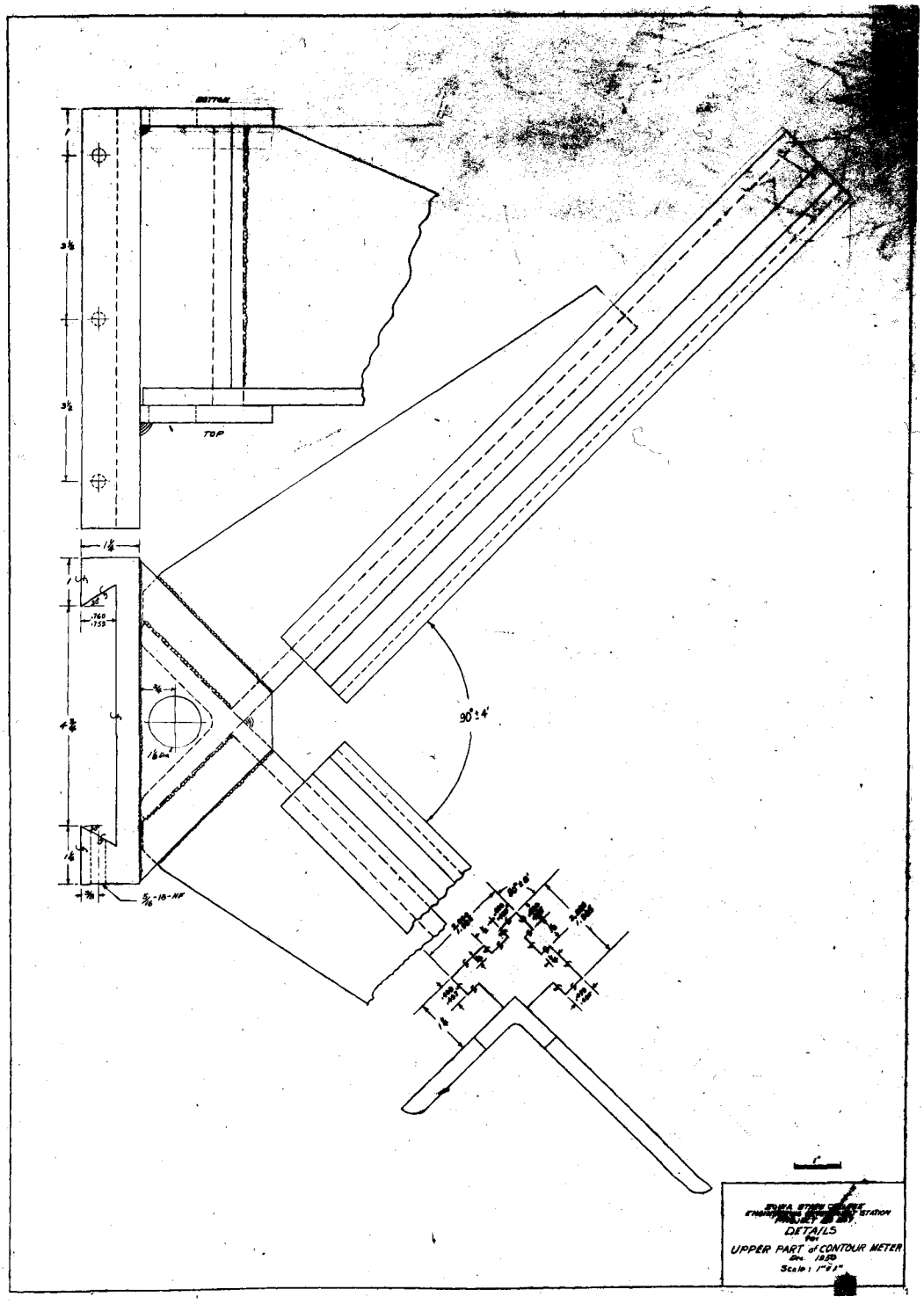


Fig. 41. Details of Sliding Assembly of Plate Contour Meter.

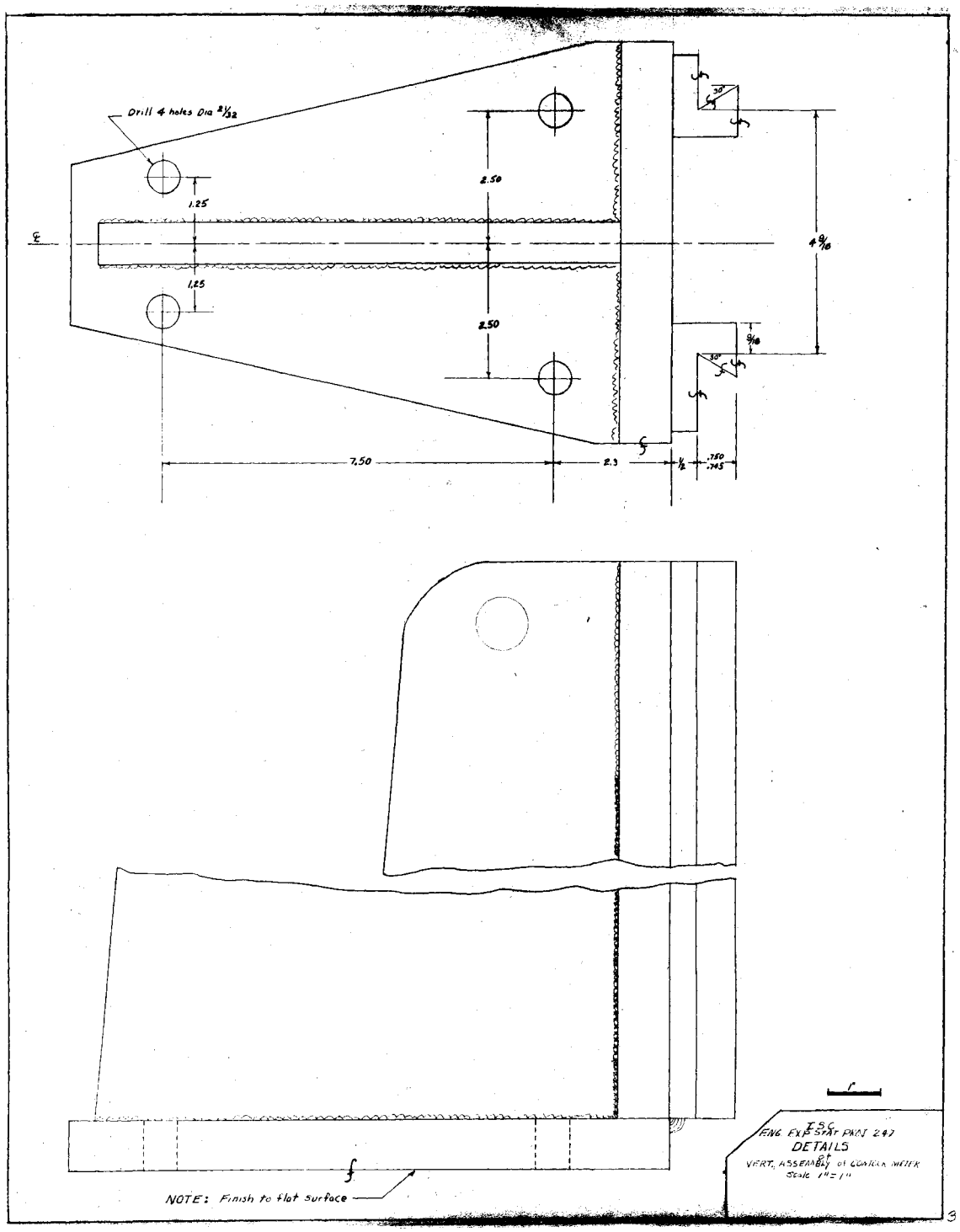


Fig. 42. Details of Vertical Assembly of Plate Contour Meter.

E. Tensile Coupon Tests

Tensile Tests

Table XIII. Properties of Tensile Test Specimens[1]

Gage	Yield Point psi	Modulus of Elasticity psi
¼"	44,000	29,875,000
5	30,300	31,350,000
8	31,600	29,450,000
10	33,200	30,300,000
14	45,000	31,250,000

[1]Each value given is an average of results from two tensile coupons.

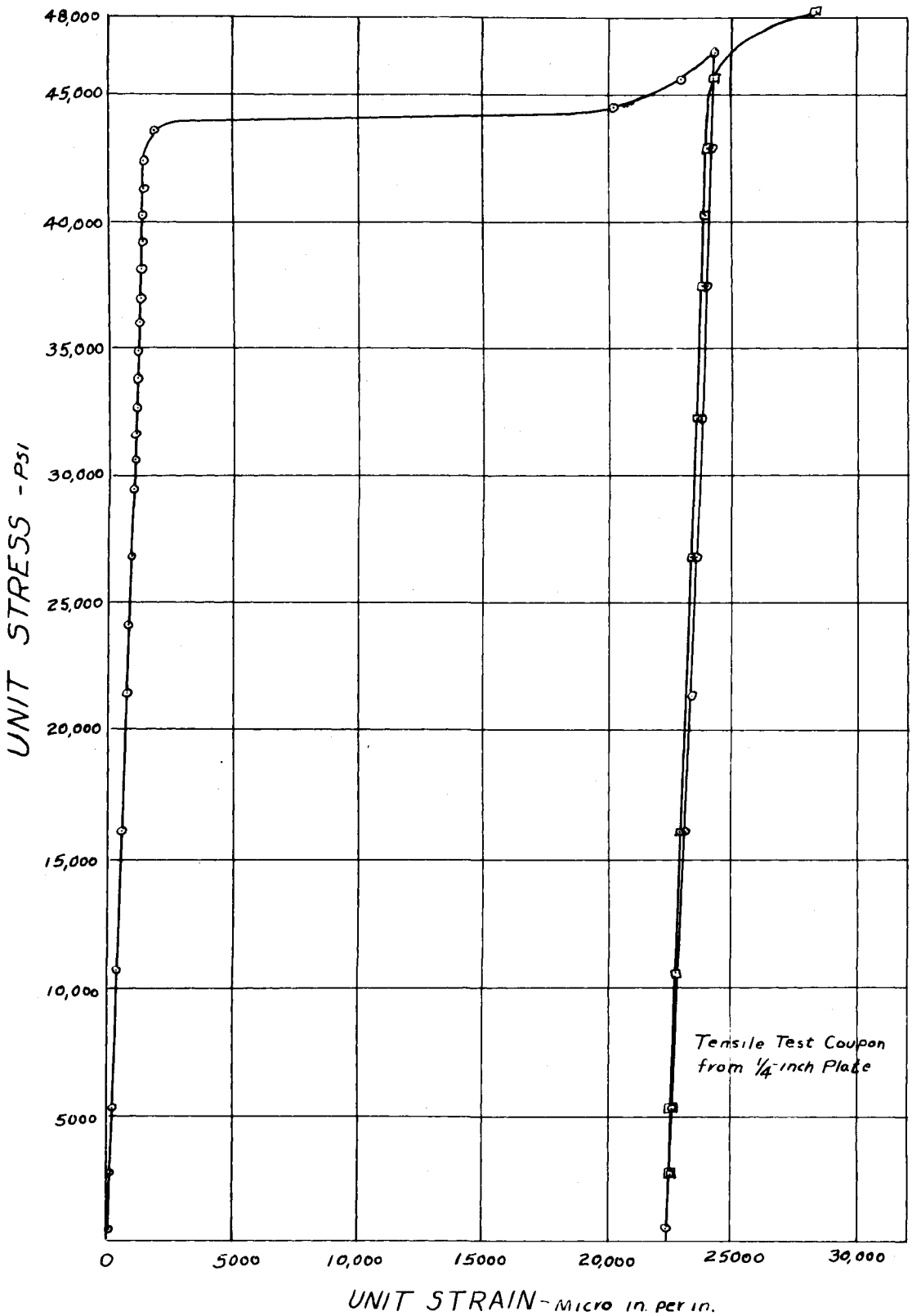


Fig. 42. Typical Stress-Strain Curve From Tensile Coupon Tests.

F. Comparison of Strain-Measuring Equipment

COMPARISON OF STRAIN MEASURING EQUIPMENT

Strain readings on the same tensile coupon under identical loading conditions were taken using three different strain-measuring methods as follows:

1. SR-4 strain scanning recorder and 48-gage automatic switching unit which were used in testing the main members.
2. SR-4 Type L strain indicator and 24-gage manual switching unit which were used in testing the tensile coupons.
3. Huggenberger tensometers.

A 1/4-inch thick tensile coupon of hot-rolled mild structural steel shaped to conform with ASTM Standards was used. One SR-4, type A-1, strain gage was mounted on either side of the coupon. The coupon was centered in a 60,000-lb. hydraulic testing machine and loaded to an average unit stress of 20,000 psi at the narrow section of the coupon, then returned to 400 psi. Each strain-measuring instrument was zeroed while a unit stress of 400 psi was maintained in the test coupon. After identical runs were made using the scanning recorder and type L strain indicator, the SR-4 strain

gages were scraped from the member and Huggenberger tensometers were mounted in the same location. An average stress of 20,000 psi was not exceeded in order to insure elastic behavior for each run.

Table XIV. Comparison of Strain Measuring Equipment

Applied Stress psi	Scanning Recorder (micro-in/in)	Type L Strain Indicator (micro-in/in)	Huggenberger Tensometers (micro-in/in)	Range of Variation Per Cent of Average
400	0	0	0	0
2,000	60	52	42	+17.0 -17.0
4,000	121	120	120	0
6,000	190	183	181	+2.7 -2.1
8,000	255	250	248	+1.6 -1.2
10,000	322	315	321	+1.0 -1.2
12,000	342	384	384	+1.3 -0.8
14,000	455	448	449	+0.9 -0.7
16,000	535	525	529	+1.1 -0.7
18,000	598	590	588	+1.0 -0.7
20,000	674	656	660	+1.7 -1.0

List of Equipment Used

1. Baldwin-Southwark SR-4 Strain Scanning Recorder.
   Serial No. D-57004.

2. Baldwin-Southwark SR-4 Automatic Switching Unit (48 Gage).
   Serial No. D-57033.

3. Baldwin-Southwark SR-4 Strain Indicator, Type L.
   Serial No. H5-9242.

4. Baldwin-Southwark Bridge Balancing Unit (24 Gage, Manual)
   Serial No. 93038-6.

5. Baldwin-Southwark AC-DC Rectifier
   Serial No. 92938-19.

6. Huggenberger Tensometers
   Serial No. ISC No. 1237 and No. 1236.

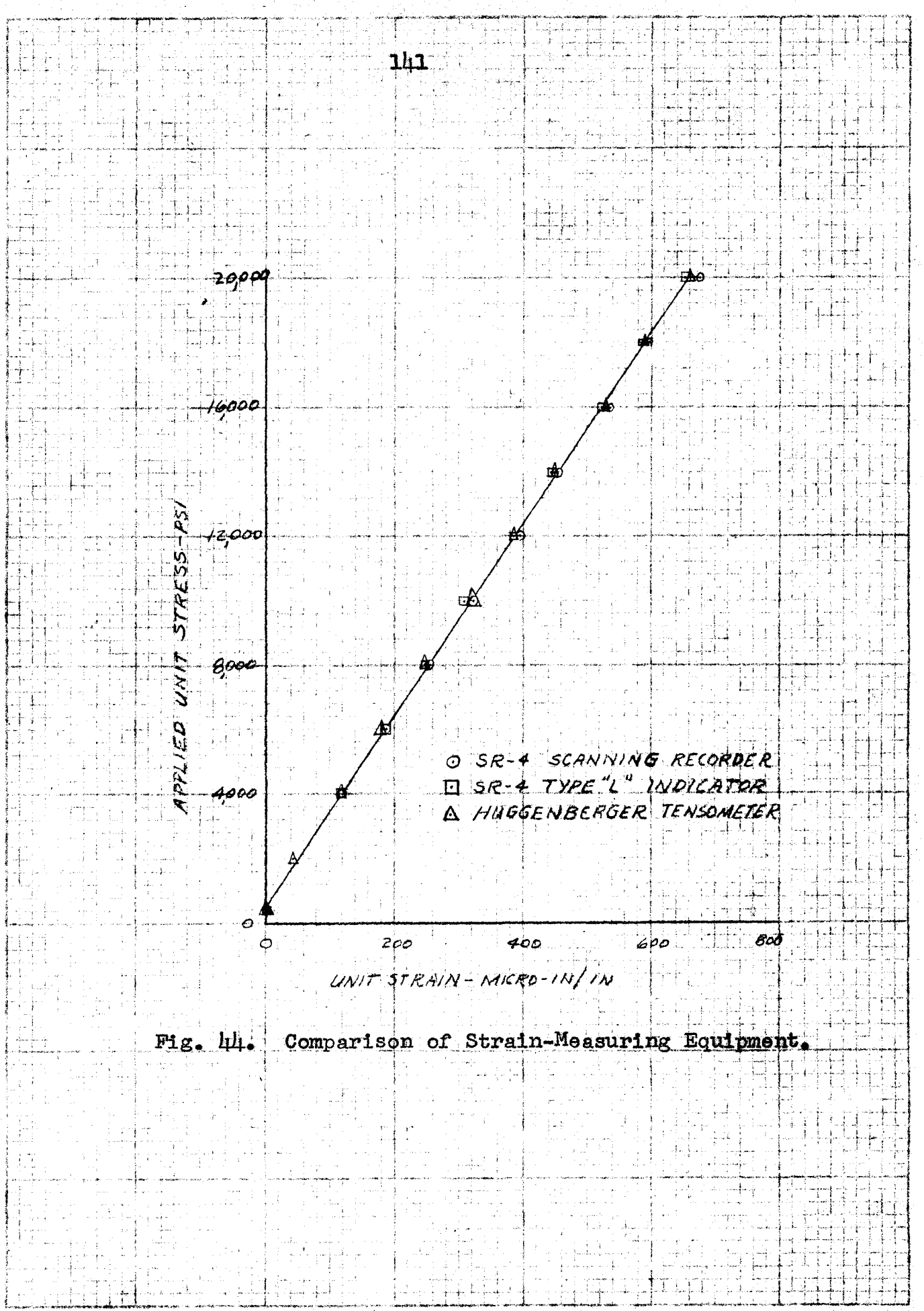


Fig. 44. Comparison of Strain-Measuring Equipment.